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Uncertainty degree and modeling of interval type-2 fuzzy sets: Definition, method and application

Chengdong Li [a,*], Guiqing Zhang [a], Jianqiang Yi [b], Ming Wang [a]

[a] School of Information and Electrical Engineering, Shandong Jianzhu University, Jinan, 250101, China

[b] Institute of Automation, Chinese Academy of Sciences, Beijing, 100190, China

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ABSTRACT

Constructing the interval type-2 fuzzy sets (IT2 FSs) models is the essential step to realize computing with words (CWW) using IT2 FSs. This paper presents a novel strategy for the modeling of symmetric IT2 FSs using their uncertainty degrees. First, we provide the definition of uncertainty degree and the properties which show the reasonability of this uncertainty measure. Also, we derive the closed-form equations of the uncertainty degrees for the most widely-used symmetric Gaussian and trapezoidal IT2 FSs. Then, based on this uncertainty measure, we present one method to construct IT2 FS models using the uncertain interval end-points data. Finally, we apply the proposed IT2 FS modeling method to a real-world application—the thermal comfort modeling problem. This application demonstrated the usefulness of the proposed method on modeling words using IT2 FSs.

1. Introduction

In real life, humans usually employ words, rather than numbers, in describing perceptions, computing and reasoning. To tackle the problems that cannot be handled through computing with numbers, Zadeh [1,2] has put forward the concept of computing with words (CWW). Since the appearance of CWW, many papers [3–8] have profoundly studied CWW and perceptual information processing using fuzzy sets (FS). Recently, Mendel [9] and Turksen [10] pointed out that it is more reasonable to adopt interval type-2 (IT2) FSs for CWW, as "words mean different things to different people". In recent years, Mendel and Wu et al. have made great contributions to CWW using IT2 FSs, from word modeling [11–16], perceptual reasoning [17–21] to the framework of perceptual computer [22–24]. For CWW using IT2 FSs, one important and essential step is to construct IT2 FS models from data or expert knowledge.

To construct IT2 FS models from data, to date, there exist two kinds of methods. The first kind is the interval approach proposed in [15] with its enhanced approach in [16]. The interval approaches adopt the statistics method to realize the IT2 FS modeling. The other one is the fuzzistics approach [11–14], which utilizes one kind of uncertainty measures of IT2 FSs—centroid to determine the parameters of IT2 FSs to ensure that the identified IT2 FS model can fit measured data in some sense. The fuzzistics approach is similar to the *method of moments* in statistics, which is an approach for parameter estimation of probability density functions by equating sample moments with unobservable moments and then solving those equations for the parameters to be estimated.

Besides the well-defined and studied centroid of IT2 FS [11–14], several other uncertainty measures of IT2 FSs have been explored. In [25], Szmidt and Kacprzyk have derived an interval cardinality for intuitionistic fuzzy sets that can be mapped to IT2 FSs. In [26], Wu and Mendel have discussed the variance and skewness of IT2 FS. In [26,27], Wu and Mendel have

* Corresponding author. Tel.: +86 531 8636 1056; fax: +86 531 8636 1567.
E-mail addresses: lichengdong@sdjzu.edu.cn, chengdong.li@ia.ac.cn (C. Li).

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defined a new interval cardinality for IT2 FS based on the representation theorem. The fuzziness or entropy of an IT2 FS has been studied by Burillo and Bustince [28], Szmidt and Kacprzyk [25], Zeng and Li [29], Vlachos and Sergiadis [30], Cornelis and Kerre [31], and Wu and Mendel [26,27].

Theoretically, all such measures of uncertainties can guide us to construct IT2 FS models from uncertain survey data. However, generally speaking, there is no closed-form method for computing these uncertainty measures. So, till now, centroid is the most commonly-used uncertainty measure for the modeling problem of IT2 FSs [11–14]. Even so, as pointed by Mendel in [14], there exist many open research issues on this topic, including: "*developing other design equations that can then be used in the interval end-points approach to solve for the FOU parameters (e.g., ones that are constrained to provide type-1 MFs if all uncertainties disappear)*"; "*developing other easier-to-use measures of uncertainty than the centroid, with the hope that they will lead to easier to obtain solutions for the interval end-points inverse problem*".

Besides the centroid, in [32], we have defined another easy-to-compute uncertainty measure. This new uncertainty measure is called uncertainty degree of the IT2 FS. It is based on the concepts of lower and upper α-cuts of an IT2 FS. In this study, we focus on utilizing this easy-to-compute uncertainty measure to modeling words using symmetric IT2 FSs. To show the reasonability, we first explore some properties of this uncertainty measure in detail. And, for some widely-used symmetric IT2 FSs, such as the Gaussian IT2 FS and the trapezoidal IT2 FSs, we derive the closed-form equations of their uncertainty degrees. Further, in order to utilize this uncertainty measure to construct symmetric IT2 FS models, we present one novel word modeling method, which contains the following steps: data collection and pre-processing, construction of the symmetric IT2 FS models, and exact and/or approximate solutions to the symmetric IT2 FS models. At last, to demonstrate the usefulness of the proposed method, a real-world application to the thermal comfort modeling is given.

The primary novelties of this work are summarized as follows.

(1) For the widely-used symmetric IT2 FSs, closed-form equations of their uncertainty degrees can be given. However, for the other uncertainty measures, it is difficult to derive such closed-form equations for these symmetric IT2 FSs. Thus, we can say that the proposed method is an easy-to-use one.
(2) The presented word modeling method is valid for symmetrical IT2 FSs with no more than four free parameters. The existing fuzzistic method is only valid for symmetrical IT2 FSs with no more than three free parameters.
(3) The proposed IT2 FS modeling method can provide type-1 (T1) FS if all uncertainties in the interval end-points data disappear. So, this new word modeling scheme can remedy the deficiency of existing methods in some sense.
(4) The proposed word modeling method is applied to a real-world application—modeling of the thermal comfort.

The rest of this paper is organized as follows. In Section 2, theoretic background of IT2 FSs is introduced. In Section 3, the uncertainty degree of IT2 FS is reviewed and its properties are proved in detail. And then, calculation of the uncertainty degrees of the widely-used symmetric IT2 FSs are provided. In Section 4, the uncertainty degree based IT2 FS modeling is studied. In Section 5, an application to thermal comfort modeling is given. Finally, conclusions are drawn in Section 6.

2. Theoretic background of IT2 FSs

This section will give a brief introduction of IT2 FSs. The detailed theoretic background of IT2 FSs can be found in [33–37].

A type-2 (T2) FS in the universal set X is a mapping from X into the set $M = [0, 1]^{[0,1]}$ of all mappings from the unit interval into itself. A T2 FS, denoted as $\tilde{A}$, can be characterized as [34,35]

$$\tilde{A} = \int_{x\in X} \mu_{\tilde{A}}(x)/x = \int_{x\in X} \left[\int_{u\in J_x} f_x(u)/u \right] \Big/ x, \quad J_x \subseteq [0, 1] \tag{1}$$

where $\int\int$ denotes union over all admissible x and u, $f_x(u)$ is the secondary membership function and J_x is the primary membership of x which is the domain of the secondary membership function.

Uncertainties in the primary memberships of a T2 FS $\tilde{A}$ consist of a bounded region that we call the footprint of uncertainty (FOU) [34,35]. The upper membership function (MF) and lower MF of the T2 FS $\tilde{A}$ are two T1 MFs that bound its FOU. The upper MF of the T2 FS $\tilde{A}$ is denoted by $\overline{\mu}_{\tilde{A}}(x)$ while its lower MF is denoted by $\underline{\mu}_{\tilde{A}}(x)$.

IT2 FSs are specific T2 FSs and can be characterized as [34,35]

$$\tilde{A} = \int_{x\in X} \left[\int_{u\in J_x} 1/u \right] \Big/ x, \quad J_x \subseteq [0, 1] \tag{2}$$

where the secondary grades of $\tilde{A}$ all equal 1.

An IT2 FS $\tilde{A}$ can be completely described by its lower and upper MFs, $\underline{\mu}_{\tilde{A}}(x)$ and $\overline{\mu}_{\tilde{A}}(x)$, respectively. The FOU of an IT2 FS is described in terms of these MFs, as

$$\mathrm{FOU}(\tilde{A}) = \bigcup_{x\in X} [\underline{\mu}_{\tilde{A}}(x), \overline{\mu}_{\tilde{A}}(x)]. \tag{3}$$