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Numerical solution for mixed convection boundary layer flow of a nanofluid along an inclined plate embedded in a porous medium

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ABSTRACT

The steady mixed convection boundary layer flow of an incompressible nanofluid along a plate inclined at an angle α in a porous medium is studied. The resulting nonlinear governing equations with associated boundary conditions are solved using an optimized, robust, extensively validated, variational finite-element method (FEM) and a finite-difference method (FDM) with a local non-similar transformation. The Nusselt number is found to decrease with increasing Brownian motion number (Nb) or thermophoresis number (Nt), whereas it increases with increasing angle α. In addition, the local Sherwood number is found to increase with a rise in Nt, whereas it is reduced with an increase in Nb and angle α. The effects of Lewis number, buoyancy ratio, and mixed convection parameter on temperature and concentration distributions are also examined in detail. The present study is of immediate interest in next-generation solar film collectors, heat-exchanger technology, material processing exploiting vertical and inclined surfaces, geothermal energy storage, and all those processes which are greatly affected by a heat-enhancement concept.

1. Introduction

A nanofluid is a fluid containing nanometer-sized particles, called nanoparticles. These fluids are engineered colloidal suspensions of nanoparticles in a base fluid. The nanoparticles used in nanofluids are typically made of metals (Al, Cu), oxides (Al_2O_3, CuO, TiO_2, SiO_2), carbides (SiC), nitrides (AlN, SiN), or nonmetals (graphite, carbon nanotubes), and the base fluid is usually a conductive fluid, such as water or ethylene glycol. Other base fluids are oil and other lubricants, bio-fluids and polymer solutions. Nanoparticles are particles that are between 1 and 100 nm in diameter. Nanofluids commonly contain up to a 5% volume fraction of nanoparticles to see effective properties over the properties of the base fluid.

Nanofluids have novel properties that make them potentially useful in many applications in heat transfer, including microelectronics, fuel cells, pharmaceutical processes, and hybrid-powered engines. They exhibit enhanced thermal conductivity and convective heat transfer coefficient compared to the base fluid. Experimental studies in the literature [1] show the typical thermal conductivity enhancements are in the range 15–40% over the base fluid, and heat transfer coefficient enhancements have been found up to 40%. Increases in thermal conductivity of this magnitude cannot be solely attributed to the higher thermal conductivity of the added nanoparticles, and there must be other mechanisms which includes particle agglomeration, nanoparticle size, volume fraction, Brownian motion, thermophoresis, particle shape/surface area, temperature and liquid layering on the nanoparticle-liquid interface, attributed to the increase in performance.

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Nomenclature

Roman

k_m	Thermal conductivity
Nu	Nusselt number
C	Nanoparticle volume fraction
C_w	Nanoparticle volume fraction on the plate
C_∞	Ambient nanoparticle volume fraction
(x, y)	Cartesian coordinates
T_w	Temperature at the plate
T_∞	Ambient temperature attained
T	Temperature on the plate
$\frac{\mathrm{Ra}_x}{\mathrm{Pe}_x}$	Mixed parameter coefficient as y tends to infinity
q_w	Wall heat flux
q_m	Wall mass flux
D_B	Brownian diffusion
D_T	Thermophoretic diffusion coefficient
$f(\eta)$	Dimensionless stream function
g	Gravitational acceleration
Nt	Thermophoresis parameter
Le	Lewis number
p	Pressure
Nb	Brownian motion parameter
w	Darcy velocity, (u, v)

Greek symbols

ρ_f	Fluid density
ρ_P	Nanoparticle mass density
ψ	Stream function
υ	Kinematic viscosity of the fluid
τ	Parameter defined by $\varepsilon(\rho c)_p/(\rho c)_f$
$(\rho c)_f$	Heat capacity of the fluid
$\phi(\eta)$	Dimensionless nanoparticle volume fraction
η	Similarity variable
$\theta(\eta)$	Dimensionless temperature
$(\rho c)_p$	Effective heat capacity of the nanoparticle material
α	Acute angle of the plate to the vertical
β	Volumetric expansion coefficient of the fluid

Subscripts

w	Condition on the plate
∞	Condition far away from the plate

Effective cooling techniques are much needed in many industries such as manufacturing, power, transportation, electronic devices and in particular the next generation of thin-film solar energy collector devices. Low thermal conductivity is a primary limitation in the development of energy-efficient heat transfer fluids. Conventional heat transfer fluids such as water, ethylene glycol, and engine oil have limited heat transfer capabilities due to their low heat transfer properties. In contrast, metals have thermal conductivities up to three times higher than these fluids, so it is naturally desirable to combine the two substances to produce a heat transfer medium that behaves like a fluid, but has the thermal properties of a metal. The term nanofluid was first proposed by Choi [2] to indicate engineered colloids composed of nanoparticles dispersed in a base fluid. The characteristic feature of nanofluids is thermal conductivity enhancement, a phenomenon observed by Masuda et al. [3].

A comprehensive survey of convective transport in nanofluids was made by Buongiorno [4] based at MIT, who considered two-phase non-homogenous model *seven slip mechanisms* that can produce a relative velocity between the nanoparticles and the base fluid: inertia, Brownian diffusion, thermophoresis, diffusiophoresis, Magnus effect, fluid drainage, and gravity. Of all of these mechanisms, only Brownian diffusion and thermophoresis were found to be important. Buongiorno's analysis [4]