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On algebraic structure of intuitionistic fuzzy soft sets ☆

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ABSTRACT

Maji et al. introduced the concept of intuitionistic fuzzy soft sets which is a generalization of fuzzy soft sets and standard soft sets. In this paper, we further discuss the operation properties and algebraic structure of intuitionistic fuzzy soft sets. The lattice structures of intuitionistic fuzzy soft sets are derived. The notions of (γ, δ)-intuitionistic fuzzy soft equalities are introduced and their basic properties are investigated. The relationships between (γ, δ)-intuitionistic fuzzy soft equalities and soft equalities introduced by Qin and Hong are developed. The notion of a mapping on intuitionistic fuzzy soft classes is introduced and several properties of the image and inverse image of intuitionistic fuzzy soft sets are presented.

1. Introduction

To solve complicated problems in economics, engineering, and environment, we cannot successfully use classical methods because of various uncertainties typical for those problems. There are three theories: theory of probability, theory of fuzzy sets, and the interval mathematics which we can consider as mathematical tools for dealing with uncertainties. But all these theories have their own difficulties. Uncertainties cannot be handled using traditional mathematical tools but may be dealt with using a wide range of existing theories such as the probability theory, the theory of (intuitionistic) fuzzy sets, the theory of vague sets, the theory of interval mathematics, and the theory of rough sets. However, all of these have their advantages as well as inherent limitations in dealing with uncertainties. One major problem shared by those theories is their incompatibility with the parameterization tools. To overcome these difficulties, Molodtsov [1] introduced the concept of soft set as a new mathematical tool for dealing with uncertainties that is free from the difficulties that have troubled the usual theoretical approaches. Molodtsov pointed out several directions for the applications of soft sets. This theory has proven useful in many different fields such as decision making [2–6], data analysis [8,7], forecasting [9] and so on.

Up to the present, research on soft sets has been very active and many important results have been achieved in the theoretical aspect. Maji et al. [10] introduced several algebraic operations in soft set theory and published a detailed theoretical study on soft sets. Ali et al. [11] further presented and investigated some new algebraic operations for soft sets. Sezgin and Atagün [12] proved that certain De Morgan's laws hold in soft set theory with respect to different operations on soft sets and discussed the basic properties of operations on soft sets such as intersection, extended intersection, restricted union and restricted difference. Kharal and Ahmad [13,14] defined the notion of a mapping on classes of (fuzzy) soft sets and studied several properties of (fuzzy) soft images and (resp., fuzzy) soft inverse images of (fuzzy) soft sets, respectively.

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Qin and Hong [15] further investigated the algebraic structure of soft sets and established the soft quotient algebra. Maji et al. [16] and Majumdar and Samanta [17] extended (classical) soft sets to fuzzy soft sets, respectively. Maji et al. [18] extended (classical) soft sets to intuitionistic fuzzy soft sets, which were further discussed in Maji et al. [19] and Jiang et al. [20]. Aktaş and Çğman [21] compared soft sets to the related concepts of fuzzy sets and rough sets. They also defined the notion of soft groups, and derived some related properties. Aygünoğlu and Aygün [22] discussed the applications of fuzzy soft sets to group theory and investigated (normal) fuzzy soft groups. Feng et al. [23] investigated soft semirings by using the soft set theory. Jun [24] introduced and investigated the notion of soft BCK/BCI-algebras. Jun and Park [25] and Jun et al. [26] discussed the applications of soft sets in ideal theory of BCK/BCI-algebras and in *d*-algebras, respectively. Koyuncu and Tanay [27] introduced and studied soft rings. Zhan and Jun [28] characterized the (implicative, positive implicative and fantastic) filteristic soft *BL*-algebras based on $\in$-soft sets and *q*-soft sets.

The purpose of this paper is to further generalize the approach introduced by Qin and Hong [15]. We focus on studying the operation properties and algebraic structure of intuitionistic fuzzy soft sets. The rest of this paper is organized as follows. Section 2 summarizes some basic concepts which will be used throughout the paper. Section 3 investigates the lattice structures of intuitionistic fuzzy soft sets. Section 4 discusses the properties of (γ, δ)-intuitionistic fuzzy soft equalities $\risingdotseq_{(\gamma,\delta)}$ and $\fallingdotseq_{(\gamma,\delta)}$. Section 4 introduces the notion of a mapping on intuitionistic fuzzy soft classes and studies the image and inverse image of intuitionistic fuzzy soft sets.

2. Preliminaries

2.1. (Intuitionistic) fuzzy sets

The theory of fuzzy sets, first developed by Zadeh in 1965 [29], provides an appropriate framework for representing and processing vague concepts by allowing partial memberships. Let X be a non-empty set. A fuzzy subset μ of X is defined as a mapping from X into [0, 1], where [0, 1] is the usual interval of real numbers. The family of all fuzzy sets of X is denoted by $\mathscr{F}(X)$. For $\mu, \nu \in \mathscr{F}(X)$, by $\mu \subseteq \nu$ we mean $\mu(x) \leq \nu(x)$ for all $x \in X$. With the min–max system proposed by Zadeh [29], fuzzy *union* and *intersection* of μ and ν, denoted by $\mu \cup \nu$ and $\mu \cap \nu$, are defined as the fuzzy subsets of X by $(\mu \cup \nu)(x) = \max\{\mu(x), \nu(x)\}$ and $(\mu \cap \nu)(x) = \min\{\mu(x), \nu(x)\}$ for all $x \in X$, respectively.

A fuzzy subset μ of X of the form

$$\mu(y) = \begin{cases} r(\neq 0) & \text{if } y = x, \\ 0 & \text{otherwise} \end{cases}$$

is said to be a *fuzzy point with support x and value r* and is denoted by x_r, where $r \in (0, 1]$.

In what follows, let $\gamma, \delta \in [0, 1]$ be such that $\gamma < \delta$. For a fuzzy point x_r and a fuzzy subset μ of X, we say that

(1) $x_r \in_\gamma \mu$ if $\mu(x) \geq r > \gamma$.
(2) $x_r\, q_\delta\, \mu$ if $\mu(x) + r > 2\delta$.
(3) $x_r \in_\gamma \vee q_\delta\, \mu$ if $x_r \in_\gamma \mu$ or $x_r\, q_\delta\, \mu$.

It is worth noting that the concepts of "$\in_\gamma$" and "q_δ" are extensions of those of "$\in$" and "q" defined in Pu and Liu [30], respectively. Let us now introduce a new ordering relation on $\mathscr{F}(X)$, denoted by "$\subseteq \vee q_{(\gamma,\delta)}$", as follows: $\forall \mu, \nu \in \mathscr{F}(X)$. By $\mu \subseteq \vee q_{(\gamma,\delta)}\, \nu$ we mean that $x_r \in_\gamma \mu$ implies $x_r \in_\gamma \vee q_\delta\, \nu$ for all $x \in X$ and $r \in (\gamma, 1]$.

In the sequel, unless otherwise stated, $\overline{\alpha}$ means α does not hold, where $\alpha \in \{\in_\gamma, q_\delta, \in_\gamma \vee q_\delta, \subseteq \vee q_{(\gamma,\delta)}\}$.

Lemma 2.1. *Let $\mu, \nu \in \mathscr{F}(X)$. Then $\mu \subseteq \vee q_{(\gamma,\delta)}\, \nu$ if and only if $\max\{\nu(x), \gamma\} \geq \min\{\mu(x), \delta\}$ for all $x \in X$.*

Proof. Assume that $\mu \subseteq \vee q_{(\gamma,\delta)}\, \nu$. Let $x \in X$. If $\max\{\nu(x), \gamma\} < \min\{\mu(x), \delta\}$, then there exists r such that $\max\{\nu(x), \gamma\} < r < \min\{\mu(x), \delta\}$. Hence $\mu(x) > r > \gamma$ and $\nu(x) < r < \delta$, implying that $x_r \in_\gamma \mu$ but $x_r\, \overline{\in_\gamma \vee q_\delta}\, \nu$, a contradiction. Therefore, $\max\{\nu(x), \gamma\} \geq \min\{\mu(x), \delta\}$.

Conversely, assume that $\max\{\nu(x), \gamma\} \geq \min\{\mu(x), \delta\}$ for all $x \in S$. If $\mu\, \overline{\subseteq \vee q_{(\gamma,\delta)}}\, \nu$, then there exists $x_r \in_\gamma \mu$ but $x_r\, \overline{\in_\gamma \vee q_\delta}\, \nu$, and so $\mu(x) \geq r$, $\nu(x) + r < 2\delta$ and $\nu(x) < r$. It follows that $\nu(x) < \delta$. Hence $\max\{\nu(x), \gamma\} < \min\{\mu(x), \delta\}$, a contradiction. Therefore, $\mu \subseteq \vee q_{(\gamma,\delta)}\, \nu$. □

Lemma 2.2. *Let $\mu, \nu, \omega \in \mathscr{F}(X)$. If $\mu \subseteq \vee q_{(\gamma,\delta)}\, \nu$ and $\nu \subseteq \vee q_{(\gamma,\delta)}\, \omega$. Then $\mu \subseteq \vee q_{(\gamma,\delta)}\, \omega$.*

Proof. It is straightforward by Lemma 2.1. □

As an important generalization of the notion of fuzzy sets, Atanassov [31] introduced the concept of an intuitionistic fuzzy set as follows.

Definition 2.3 (*[31]*)**.** An intuitionistic fuzzy set A of a non-empty set X is an object having the form

$$A = \{\langle x, \mu_A(x), \lambda_A(x)\rangle | x \in X\}$$

where the functions $\mu_A : X \to [0, 1]$ and $\lambda_A : X \to [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\lambda_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \lambda_A(x) \leq 1$ for all $x \in X$.