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A trust region SQP-filter method for nonlinear second-order cone programming[☆]

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ABSTRACT

In this paper, we consider the nonlinear second-order cone programming problem. By combining an SQP method and filter technique, we present a trust region SQP-filter method for solving this problem. The proposed algorithm avoids using the classical merit function with penalty term. Furthermore, under standard assumptions, we prove that the iterative sequence generated by the presented algorithm converges globally. Preliminary numerical results indicate that the algorithm is promising.

1. Introduction

The nonlinear second-order cone programming (NSOCP for short) problem is stated as follows:

$$\begin{aligned} &\min f(x), \\ \text{s.t.}\quad &h(x) \in K. \end{aligned} \tag{1}$$

where $f : R^n \to R$, $h : R^n \to R^l$ are twice continuously differentiable functions, K is the Cartesian product of second-order cones, that is $K = K^{l_1} \times K^{l_2} \times \cdots \times K^{l_s}$ with $l_1 + l_2 + \cdots + l_s = n$, and the l_i-dimensional second-order cone K^{l_i} is defined by

$$K^{l_i} := \{(x_1, x_2^T)^T \in R \times R^{l_i-1} | x_1 \geq \|x_2\|\},$$

with $\|\cdot\|$ denoting the Euclidean norm and K^1 denoting the set of nonnegative reals R_+ (the nonnegative orthant in R).

The second-order cone programming problem has a wide range of applications in many fields, such as engineering, control and so on [1–7]. As a special case, the linear second-order cone programming should be mentioned, which is to find a vector $x \in R^n$ such that

$$\min \left\{ \sum_{i=1}^{n} c_i^T x_i : \sum_{i=1}^{n} A_i^T x_i = b, x_i \in K_i, i = 1, 2, \ldots, n \right\}. \tag{2}$$

Some algorithms have been developed to treat this class of problems [8,9,2,10,11], but there is little work for solving nonlinear second-order cone programming. It is well known that the sequential quadratic programming (SQP) method is a

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classical and efficient approach for solving nonlinear programming and has been extensively discussed in [12–18], as well as numerous subsequent references. Lately, Kato and Fukushima proposed an SQP-type algorithm for nonlinear second-order cone programming [19]. Their method is to generate iteratively a sequence $\{x^k\}$ which converges to a Karush–Kuhn–Tucker point of the problem (1) by solving the following quadratic subproblem $QP(x^k, d)$:

$$\begin{aligned} \min \quad & s_k(d) = \nabla f(x^k)^T d + \frac{1}{2} d^T M_k d \\ \text{s.t.} \quad & h(x^k) + \nabla h(x^k)^T d \in K, \end{aligned} \tag{3}$$

where x^k is a current iterate and M_k is a symmetric positive definite matrix approximating the Hessian of the Lagrangian function of problem (1) in some sense. It is obvious that the subproblem (3) is a convex programming problem.

However, whether the exact linear search or the inexact linear search is used in the general SQP method, there are many difficulties obtaining the penalty parameter in the penalty function, as we see that the algorithm for nonlinear second-order cone programming presented by Kato et al. [19] also needs to choose the penalty parameter. In order to dispense with the idea of a penalty function in linear search and overcome the infeasibility of the general QP subproblem, we consider a modified SQP-filter method to solve the nonlinear second-order cone programming problem.

Ever since filter methods were introduced for constrained optimization by Fletcher and Leyffer [20,21], they have attached a lot of attention [22–26] partially due to their superior numerical results, more importantly, they avoid some pitfalls of penalty function methods. The filter methods give up the strict monotone behavior of usual measures, like penalty functions. Instead of combining the objective and constraint violation into a single function, they view (1) as a biobjective optimization problem that minimizes $f(x)$ and constraint violation, and then the filter allows to increase the flexibility in optimization processes to accept new iterates and generally allows larger steps towards the solution. More recently, they have been extended to deal with many different optimization problems, such as a pattern search algorithm for derivative-free optimization [22], a bundle method for non-smooth optimization [27], and a trust region filter method for general nonlinear programming [24], and so on.

In this paper, we present a trust region SQP-filter method for nonlinear second-order cone programming by considering the merits of the modified SQP method and filter technique. It is shown that our algorithm has the following good properties:

(1) It does not need to consider the penalty parameter, avoiding therefore the update of penalty parameters associated with the penalization of the constraints in merit functions;
(2) The algorithm either terminates at a Karush–Kuhn–Tucker (KKT) point within finite steps or generates an infinite sequential whose every accumulation point is a KKT point under proper conditions.

The rest of the paper is organized as follows. In the next section, we first review some preliminaries associated with second-order cones, then give some notations and lemmas to develop the modified SQP method, and finally study the filter technique. In Section 3, we present a trust region SQP-filter algorithm for nonlinear second-order cone programming. The global convergence of the algorithm is discussed in Section 4. Preliminary numerical results are reported in Section 5. Some conclusions are given in Section 6.

Throughout this paper, all vectors are column vectors, and ^{T} denotes transpose. I represents an identity matrix of suitable dimension, and $\|\cdot\|$ denotes the Euclidean norm defined by $\|x\| := \sqrt{x^T x}$ for a vector x. For any differentiable function $f : R^n \to R^n$, $\nabla f(x)$ denotes the gradient of f at x. Let int K denote the interior of K. $x \succeq y$ or $x \succ y$ means that $x - y \in K$ or $x - y \in \text{int } K$, respectively. R_{++} means the positive orthant of R. For simplicity, we use $x = (x_1, x_2) \in R \times R^{n-1}$ for the column vector $x = (x_1, x_2^T)^T$.

2. Preliminaries

In this section, we first review some basic facts on Euclidean Jordan algebra with the second-order cone. Then we give some notations and lemmas for QP problems. Finally, we shall recall the filter technology.

Euclidean Jordan algebra has been introduced in [8,28], which provides a useful methodology of dealing with SOC.

A Euclidean Jordan algebra is a triple $(V, \langle\cdot,\cdot\rangle, \circ)$ (V for short), where $(V, \langle\cdot,\cdot\rangle)$ is a finite dimensional inner product space over R and $(x, y) \mapsto x \circ y : V \times V \to V$ is a bilinear mapping which satisfies the following conditions:

(a) $x \circ y = y \circ x$, for any $x, y \in V$.
(b) $x \circ (x^2 \circ y) = x^2 \circ (x \circ y)$ for all $x, y \in V$ where $x^2 = x \circ x$.
(c) $\langle (x \circ y, z) \rangle = \langle (x, y \circ z) \rangle$ for all $x, y, z \in V$.

Based on the general definition of Euclidean Jordan algebra, given the n-dimension Euclidean space R^n, the inner product and the Jordan product are defined as follows respectively.

For any $x = (x_1, \bar{x}_2) \in R \times R^{n-1}$, $y = (y_1, \bar{y}_2) \in R \times R^{n-1}$, define the Jordan product:

$$x \circ y := (x^T y, x_1 \bar{y}_2 + y_1 \bar{x}_2).$$

the inner product is:

$$\langle x, y \rangle = \sum_{i=1}^{n} x_i y_i,$$