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Stochastic flow networks via multiple paths under time threshold and budget constraint

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ABSTRACT

This paper extends the quickest path problem to a stochastic flow network in which the capacity of each arc is variable. We mainly evaluate the system reliability that d units of data can be sent from the source to the sink under both time threshold T and budget B. In particular, the data are transmitted through multiple disjoint minimal paths simultaneously in order to reduce the transmission time. A simple algorithm is proposed to generate all lower boundary points for (d, T, B), and the system reliability can then be computed in terms of such points by utilizing a union of subsets. Computational complexity in both worst case and average cases show that the proposed algorithm can be executed efficiently.

1. Introduction

For a network in which each arc has the length attribute, the shortest path problem is one of the well-known and practical problems in computer science, operations research, networking and other areas. When goods or commodities are transmitted from the source node to the sink node through a flow network, it is desirable to adopt the shortest path, least cost path, largest capacity path, shortest delay path, or some combination of multiple criteria [1–4], which are all variants of the shortest path problem. From the viewpoint of quality management, it is an essential issue to reduce the transmission time through the time-based network, especially through computer and telecommunication networks. Hence, a version of the shortest path problem called the quickest path problem [5] arises to find a path (named the quickest path) with minimum transmission time for sending a given amount of data from the source to the sink, where each arc has two attributes; the capacity and the lead time [5–8]. More specifically, the capacity and the lead time are both assumed to be deterministic. Since then, several variants of quickest path problems are proposed; constrained quickest path problem [9,10], the first k quickest paths problem [11–14], and all-pairs quickest path problem [15].

For many modern flow networks such as computer networks, telecommunication networks, urban traffic networks, logistics networks, etc., the arc's capacity should be variable due to failure, partial failure, maintenance, etc. Such a network is the so-called a stochastic flow network [16–24] in which each arc has several possible capacities or states. It is noteworthy that no time attribute is considered in the previous literatures. The purpose of this paper is mainly to extend the quickest path problem to a general case that the data are transmitted through multiple disjoint minimal paths (MPs) simultaneously to reduce the transmission time, where an MP is an ordered sequence of arcs from the source to the sink without loops. We model a stochastic flow network with a time attribute to focus on the transmission time. For convenience, this paper first concentrates on the transmission time through two MPs. The case to transmit the data through more than two MPs can be extended easily. Besides, cost is another crucial factor in enterprise competing. The budget constraint is thus included in our problem. We evaluate the probability that the stochastic flow network can send d units of data from the source to the

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sink under both time threshold T and budget B. Such a probability is named the system reliability throughout this paper. Algorithm I is first proposed in Section 3 to generate all lower boundary points for (d, T, B), the minimal system states fulfilling the requirements. The system reliability may be calculated in terms of such points by utilizing a union of subsets. An illustrative example in different cases is presented in Section 4. Computational complexity analysis of the algorithm is shown in Section 5. The extension to a more than 2 MPs case is demonstrated in Section 6.

2. Problem modeling

Let $G = (N, A, L, M, C)$ denote a stochastic flow network with a source and a sink where N denotes the set of nodes, $A = \{a_i|1 \le i \le n\}$ denotes the set of arcs, $L = \{l_i|1 \le i \le n\}$ with l_i denoting the lead time of a_i, $M = \{m_i|1 \le i \le n\}$ with m_i denoting the maximal capacity of a_i, and $C = \{c_i|1 \le i \le n\}$ with c_i denoting the transmission cost on a_i. The capacity is the maximal number of data sent through the medium (an arc or an MP) per unit of time. The transmission cost is counted by each unit of flow. The (current) capacity of arc a_i, denoted by x_i, takes possible value $0 = b_{i1} < b_{i2} < \cdots < b_{ir_i} = m_i$, where b_{ij} is an integer for $j = 1, 2, \dots, r_i$. The vector $X = (x_1, x_2, \dots, x_n)$ is called the system state of G. Such a G is assumed to further satisfy the following assumptions:

1. All data are sent through two MPs simultaneously.
2. Each node is perfectly reliable.
3. The capacity of each arc is a random variable with a given probability distribution.
4. The capacities of different arcs are statistically independent.

Suppose there are m MPs; $P_1, P_2, \dots, P_m$. For each MP $P_j = \{a_{j1}, a_{j2}, \dots, a_{jn_j}\},\ j = 1, 2, \dots, m$, the capacity of P_j under the system state X is $\min_{1\le k\le n_j}(x_{jk})$. If d units of data are transmitted only through P_j, then the total cost $F(d, P_j)$ is

$$F(d, P_j) = \sum_{i=1}^{n_j}(d \cdot c_{ji}), \tag{1}$$

where $(d.c_{ji})$ is the total cost through a_{ji} for $1 \le i \le n_j$. On the other hand, the transmission time to send d units of data through P_j under the system state X, denoted by $\lambda(d, X, P_j)$, is

$$\text{lead time of } P_j + \left\lceil \frac{d}{\text{the capacity of } P_j} \right\rceil = \sum_{k=1}^{n_j} l_{jk} + \left\lceil \frac{d}{\min\limits_{1\le k\le n_j} x_{jk}} \right\rceil, \tag{2}$$

where $\lceil x \rceil$ is the smallest integer such that $\lceil x \rceil \ge x$. It contradicts the time threshold if $\lambda(d, X, P_j) > T$. We have the result of Lemma 1.

Lemma 1. *$\lambda(d, X, P_j) \ge \lambda(d, Y, P_j)$ if $X < Y$.*

Proof. If $X < Y$, then $x_{jk} \le y_{jk}$ for each $a_{jk} \in P_j$ and $\min_{1\le k\le n_j} x_{jk} \le \min_{1\le k\le n_j} y_{jk}$. Thus, $\left\lceil \frac{d}{\min_{1\le k\le n_j} x_{jk}} \right\rceil \ge \left\lceil \frac{d}{\min_{1\le k\le n_j} y_{jk}} \right\rceil$ and $\lambda(d, X, P_j) \ge \lambda(d, Y, P_j)$. □

Let d_1 and d_2 be the assigned demands through the first and second MP, respectively. Without loss of generality, say P_1 and P_2. The following equation states that the total cost under both P_1 and P_2 cannot exceed the budget,

$$F(d_1, P_1) + F(d_2, P_2) \le B. \tag{3}$$

For convenience, let $\Gamma = \{(d_1, d_2)|(d_1, d_2)\text{satisfies constraint (3)}\}$. The notation $\varphi(d_1, d_2, X, B)$ denotes the minimum transmission time to send d_1 and d_2 units of data through P_1 and P_2, respectively, under both the system state X and budget B. For the demand pair $(d_1, d_2) \in \Gamma$,

$$\varphi(d_1, d_2, X, B) = \max\{\lambda(d_1, X, P_1), \lambda(d_2, X, P_2)\}. \tag{4}$$

Moreover, let $\theta(d, X, B)$ denote the minimum transmission time to send d units of data from the source to the sink under both the system state X and budget B, then

$$\theta(d, X, B) = \min_{(d_1,d_2)\in\Gamma}\{\varphi(d_1, d_2, X, B)\}. \tag{5}$$

The system reliability $R_{d,T,B}$ to meet both the time threshold and budget constraint is thus $\Pr\{X|\theta(d, X, B) \le T\}$. Any system state X with $\theta(d, X, B) \le T$ means that X can send d units of data from the source to the sink under both time threshold T and budget B. Let Φ be the set of such X, and $\Phi_{\min} = \{X|X \text{ is minimal in } \Phi\}$, where $Y \ge X$ if $y_i \ge x_i$ for $i = 1, 2, \dots, n$, and $Y > X$ if $Y \ge X$ and $y_i > x_i$ for at least one i. Then $X \in \Phi_{\min}$ is called a lower boundary point for (d, T, B) throughout this paper. Equivalently, X is a lower boundary point for (d, T, B) if and only if (i) $\theta(d, X, B) \le T$ and (ii) $\theta(d, Y, B) > T$ for any system state Y with $Y < X$. Hence, we have the following lemma to revise the system reliability equation.