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# Applications of the operator $H(\alpha, \beta)$ to the Humbert double hypergeometric functions

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ABSTRACT

By making use of some techniques based upon certain new inverse pairs of symbolic operators, the authors investigate several decomposition formulas associated with Humbert hypergeometric functions $\Phi_1$, $\Phi_2$, $\Phi_3$, $\Psi_1$, $\Psi_2$, $\Xi_1$ and $\Xi_2$. These operational representations are constructed and applied in order to derive the corresponding decomposition formulas. With the help of these inverse pairs of symbolic operators, as many as 34 decomposition formulas are found. Euler type integrals connected with Humbert functions are also presented.

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## 1. Introduction

A great interest in the theory of hypergeometric functions (that is, hypergeometric functions of several variables) is motivated essentially by the fact that the solutions of many applied problems involving (for example) partial differential equations are obtainable with the help of such hypergeometric functions (see, for details, [1, p. 47]; see also the recent works [2–6] and the references cited therein). For instance, the energy absorbed by some nonferromagnetic conductor sphere included in an internal magnetic field can be calculated with the help of such functions [7,8]. Hypergeometric functions of several variables are used in physical and quantum chemical applications as well [9–11]. Especially, many problems in gas dynamics lead to those of degenerate second-order partial differential equations, which are then solvable in terms of multiple hypergeometric functions. Among examples, we can cite the problem of adiabatic flat-parallel gas flow without whirlwind, the flow problem of supersonic current from vessel with flat walls, and a number of other problems connected with gas flow [12,13]. It is noted that Riemann's functions and fundamental solutions of the degenerate second-order partial differential equations are expressible by means of hypergeometric functions of several variables [2–4]. In the investigation of the boundary value problems for these partial differential equations, we need decompositions for hypergeometric functions of several variables in terms of simpler hypergeometric functions of the Gauss and Humbert types. We recall the Humbert functions $\Phi_1$, $\Phi_2$, $\Phi_3$, $\Psi_1$, $\Psi_2$, $\Xi_1$, and $\Xi_2$ in two variables defined by (see [14, p. 126])

$$\Phi_1(\alpha, \beta; \gamma; x, y) = \sum_{m,n=0}^{\infty} \frac{(\alpha)_{m+n}(\beta)_m}{(\gamma)_{m+n} m! n!} x^m y^n \quad (|x| < 1, \ |y| < \infty), \tag{1.1}$$

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$$\Phi_2(\beta_1,\beta_2;\gamma;x,y)=\sum_{m,n=0}^{\infty}\frac{(\beta_1)_m(\beta_2)_n}{(\gamma)_{m+n}m!n!}x^my^n\quad(|x|<\infty,\ |y|<\infty),\tag{1.2}$$

$$\Phi_3(\beta;\gamma;x,y)=\sum_{m,n=0}^{\infty}\frac{(\beta)_m}{(\gamma)_{m+n}m!n!}x^my^n\quad(|x|<\infty,\ |y|<\infty),\tag{1.3}$$

$$\Psi_1(\alpha,\beta;\gamma_1,\gamma_2;x,y)=\sum_{m,n=0}^{\infty}\frac{(\alpha)_{m+n}(\beta)_m}{(\gamma_1)_m(\gamma_2)_nm!n!}x^my^n\quad(|x|<1,\ |y|<\infty),\tag{1.4}$$

$$\Psi_2(\alpha;\gamma_1,\gamma_2;x,y)=\sum_{m,n=0}^{\infty}\frac{(\alpha)_{m+n}}{(\gamma_1)_m(\gamma_2)_nm!n!}x^my^n\quad(|x|<\infty,\ |y|<\infty),\tag{1.5}$$

$$\Xi_1(\alpha_1,\alpha_2,\beta;\gamma;x,y)=\sum_{m,n=0}^{\infty}\frac{(\alpha_1)_m(\alpha_2)_n(\beta)_m}{(\gamma)_{m+n}m!n!}x^my^n\quad(|x|<1,\ |y|<\infty),\tag{1.6}$$

$$\Xi_2(\alpha,\beta;\gamma;x,y)=\sum_{m,n=0}^{\infty}\frac{(\alpha)_m(\beta)_m}{(\gamma)_{m+n}m!n!}x^my^n\quad(|x|<1,\ |y|<\infty),\tag{1.7}$$

where $(\alpha)_m=\Gamma(\alpha+m)/\Gamma(\alpha)$ is the Pochhammer symbol. For various multivariable hypergeometric functions including the Lauricella multivariable functions $F_A^{(r)}$, $F_B^{(r)}$, $F_C^{(r)}$ and $F_D^{(r)}$, Hasanov and Srivastava [15,16] presented a number of decomposition formulas in terms of such simpler hypergeometric functions as the Gauss and Appell functions. The main object of this sequel to the works of Hasanov and Srivastava [15,16] is to show how some rather elementary techniques based upon certain inverse pairs of symbolic operators would lead us easily to several decomposition formulas associated with the Humbert hypergeometric functions $\Phi_1$, $\Phi_2$, $\Phi_3$, $\Psi_1$, $\Psi_2$, $\Xi_1$ and $\Xi_2$. Over six decades ago, Burchnall and Chaundy [17,18] and Chaundy [19] systematically presented a number of expansion and decomposition formulas for some double hypergeometric functions in series of simpler hypergeometric functions. Their method is based upon the following inverse pairs of symbolic operators:

$$\nabla_{xy}(h):=\frac{\Gamma(h)\Gamma(\delta_1+\delta_2+h)}{\Gamma(\delta_1+h)\Gamma(\delta_2+h)}=\sum_{k=0}^{\infty}\frac{(-\delta_1)_k(-\delta_2)_k}{(h)_kk!},\tag{1.8}$$

$$\begin{aligned}\Delta_{xy}(h)\ &:=\ \frac{\Gamma(\delta_1+h)\Gamma(\delta_2+h)}{\Gamma(h)\Gamma(\delta_1+\delta_2+h)}=\sum_{k=0}^{\infty}\frac{(-\delta_1)_k(-\delta_2)_k}{(1-h-\delta_1-\delta_2)_kk!}\\&=\ \sum_{k=0}^{\infty}\frac{(-1)^k(h)_{2k}(-\delta_1)_k(-\delta_2)_k}{(h+k-1)_k(h+\delta_1)_k(h+\delta_2)_kk!},\end{aligned}\tag{1.9}$$

and

$$\begin{aligned}\nabla_{xy}(h)\Delta_{xy}(g)\ &:=\ \frac{\Gamma(h)\Gamma(\delta_1+\delta_2+h)}{\Gamma(\delta_1+h)\Gamma(\delta_2+h)}\frac{\Gamma(\delta_1+g)\Gamma(\delta_2+g)}{\Gamma(g)\Gamma(\delta_1+\delta_2+g)}\\&=\ \sum_{k=0}^{\infty}\frac{(g-h)_k(g)_{2k}(-\delta_1)_k(-\delta_2)_k}{(g+k-1)_k(g+\delta_1)_k(g+\delta_2)_kk!}\\&=\ \sum_{k=0}^{\infty}\frac{(h-g)_k(-\delta_1)_k(-\delta_2)_k}{(h)_k(1-g-\delta_1-\delta_2)_kk!}\left(\delta_1:=x\frac{\partial}{\partial x};\ \delta_2:=y\frac{\partial}{\partial y}\right).\end{aligned}\tag{1.10}$$

We introduce the following multivariable symbolic operators:

$$H_{x_1,\ldots,x_l}(\alpha,\beta):=\frac{\Gamma(\beta)\Gamma(\alpha+\delta_1+\cdots+\delta_l)}{\Gamma(\alpha)\Gamma(\beta+\delta_1+\cdots+\delta_l)}=\sum_{k_1,\ldots,k_l=0}^{\infty}\frac{(\beta-\alpha)_{k_1+\cdots+k_l}(-\delta_1)_{k_1}\cdots(-\delta_l)_{k_l}}{(\beta)_{k_1+\cdots+k_l}k_1!\cdots k_l!}\tag{1.11}$$

and

$$\bar{H}_{x_1,\ldots,x_l}(\alpha,\beta):=\frac{\Gamma(\alpha)\Gamma(\beta+\delta_1+\cdots+\delta_l)}{\Gamma(\beta)\Gamma(\alpha+\delta_1+\cdots+\delta_l)}=\sum_{k_1,\ldots k_l=0}^{\infty}\frac{(\beta-\alpha)_{k_1+\cdots+k_l}(-\delta_1)_{k_1}\cdots(-\delta_l)_{k_l}}{(1-\alpha-\delta_1-\cdots-\delta_l)_{k_1+\cdots+k_l}k_1!\cdots k_l!}\tag{1.12}$$

$$\left(\delta_j:=x_j\frac{\partial}{\partial x_j},\ j=1,\ldots,l;\ l\in\mathbb{N}:=\{1,2,3,\ldots\}\right).$$

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