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Stability and bifurcation in a diffusive Lotka–Volterra system with delay[☆]

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ABSTRACT

In this paper, we investigate the dynamics of a class of diffusive Lotka–Volterra equation with time delay subject to the homogeneous Dirichlet boundary condition in a bounded domain. The existence of spatially nonhomogeneous steady state solution is investigated by applying Lyapunov–Schmidt reduction. The stability and nonexistence of Hopf bifurcation at the spatially nonhomogeneous steady-state solution with the changes of a specific parameter are obtained by analyzing the distribution of the eigenvalues. Moreover, we illustrate our general results by applications to models with a single delay and one-dimensional spatial domain.

1. Introduction

To describe the interactions between the species, some first-order ordinary differential equations (ODEs) of predator–prey type were formulated by American chemist and biologist Alfred James Lotka in 1920 [1], and Italian mathematician Vito Volterra in 1926 [2]. The simplest form of these equations is

$$\begin{cases} \dfrac{\mathrm{d}u}{\mathrm{d}t} = lu - Buv, \\ \dfrac{\mathrm{d}v}{\mathrm{d}t} = -mv + Cuv, \end{cases} \tag{1.1}$$

which was regarded as a prototypical predator–prey system in the ecological studies. The coefficients l, m, B, C are positive constants, where l is the intrinsic growth rate of the prey; m is the death rate of the predator; B and C represent the strength of the relative effect of the interaction on the two species.

Later, in order to make the model adapt to a wider species and reflect the interaction more accurately, Volterra [3] modified system (1.1) as

$$\begin{cases} \dfrac{\mathrm{d}u}{\mathrm{d}t} = lu - Au^2 - Buv, \\ \dfrac{\mathrm{d}v}{\mathrm{d}t} = mv - Dv^2 + Cuv, \end{cases} \tag{1.2}$$

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where $l, A, B, C, D \geq 0$, and $m \in \mathbb{R}$. Here, l, B, C have the same meaning as system (1.1), m represents the death rate of the predator (respectively, the intrinsic growth rate of the predator) if $m < 0$ (respectively, $m > 0$), and A measures the strength of competition among individuals of species u, D has a similar meaning to A.

The Lotka–Volterra predator–prey model (1.2) possesses a unique coexistence steady-state solution (u_*, v_*) when $D \geq 0$, and (u_*, v_*) is globally asymptotically stable when $D > 0$ (a complete phase plane analysis of (1.2) can be found in, for example, [4] or [5]). Biologists have observed that some predator–prey interactions lead to a system with a stable equilibrium, however, some others do not, and there exists an ecological process which destabilizes the coexisting equilibrium. In view of the limited ability of a predator to consume its prey, a general functional response of the predator $\phi(u)$ was introduced by Solomon [6] and Holling [7,8] so that the classical Lotka–Volterra model is modified as

$$\begin{cases} \dfrac{\mathrm{d}u}{\mathrm{d}t} = lu - Au^2 - B\phi(u)v, \\ \dfrac{\mathrm{d}v}{\mathrm{d}t} = mv - Dv^2 + C\phi(u)v. \end{cases} \tag{1.3}$$

Here $\phi(u)$ is a positive and nondecreasing function of u (prey density). Note that system (1.2) is a special case of (1.3) when $\phi(u) = u$, and that $\phi(u)$ may have many possible choices such as the Holling type-II functional response which is most commonly used in the ecological literature and is defined by

$$\phi(u) = \frac{u}{1 + ku},$$

where k is a positive constant measuring the ability of a generic predator to kill and consume a generic prey. When $m < 0$ and $D = 0$, system (1.3) becomes the Rosenzweig–MacArthur model, which is widely used in real-life ecological applications [9]; when $m < 0$ and $D > 0$, the model was used by Bazykin [10,11].

In the aforementioned literature, the functional response $\phi(u)$ is assumed to be governed by a principle of causality, that is, the future state of the functional response of the predator $\phi(u)$ is independent of the past states and is determined solely by the present. However, a realistic model would include some of past states of the functional response of the predator, one can refer to [12–17]. Based on these motivations, much attention has been drawn to study the functional response of the predator with delay effect, i.e., replacing $\phi(u(t, x))$ in the second equation of (1.3) by $\phi(u(t - \tau, x))$, is better to reflect the interactions exactly than the functional response without delay effect.

On the other hand, the spatial component of ecological interactions has been identified as an important factor in how ecological communities are shaped, and understanding the role of space is challenging both theoretically and empirically [18–22]. Recently the studies on problems of the delayed reaction–diffusion equations have attracted many researchers' attention over the given spatial domain, one can refer to [23–25,13,14,26–36] and the references therein. In this paper, we will study the following reaction–diffusion predator–prey model with delay effect under the Dirichlet boundary condition

$$\begin{cases} \dfrac{\partial u(t,x)}{\partial t} = d\Delta u(t,x) + lu(t,x) - Au^2(t,x) - B\phi(u(t,x))v(t,x), \quad x \in \Omega,\ t \geq 0, \\ \dfrac{\partial v(t,x)}{\partial t} = d\Delta v(t,x) + mv(t,x) - Dv^2(t,x) + C\phi(u(t-\tau,x))v(t,x), \quad x \in \Omega,\ t \geq 0, \\ u(t,x) = v(t,x) = 0, \quad x \in \partial\Omega,\ t \geq 0, \end{cases} \tag{1.4}$$

where A, B, C, and D are strictly positive, Δ denotes the Laplacian operator, d is the diffusion coefficient. In biological terms, u and v can be interpreted as the densities of prey and predator populations, respectively, and A, D self-limitation constants. In the absence of predators and the diffusion effect, the prey species follows the logistic equation $\frac{\mathrm{d}u}{\mathrm{d}t} = u[l - Au]$, with l being the intrinsic growth rate of the prey and A being strictly positive for species with self-limitation. In the presence of predators and the diffusion effect, there is a hunting term $B\phi(u)$ $(B > 0)$, where ϕ is a positive and nondecreasing function. In the absence of prey species and the diffusion effect, the predator species also follows the logistic equation $\frac{\mathrm{d}v}{\mathrm{d}t} = v[m - Dv]$, with m being the intrinsic growth rate of the predator and D being strictly positive for species with self-limitation. The positive feedback $C\phi(u(t - \tau))$ has a positive delay τ which represents the time of the predator maturation.

Many researchers have been concentrating on the dynamics of spatially homogeneous steady state solutions of diffusive systems, for example, Faria [37] considered a delay diffusive predator–prey system with one delay and a unique positive homogeneous steady state solution E^*. Faria [37] studied the local stability of E^* and described the Hopf bifurcation which occurs as the delay (taken as a parameter) crosses some critical values. Yi et al. [38] investigated a homogeneous reaction–diffusion model describing the control growth of mammalian hair, and found that when one of the dimensionless parameter is less than one, the unique positive homogeneous steady-state solution is globally asymptotically stable. Li and Li [39] studied a stage-structured predator–prey system with Holling type-III functional response and time delay due to the gestation of the predator and obtained the existence of a Hopf bifurcation at the homogeneous steady-state solution.

Recently, Kuto and Tsujikawa [40] employed Leray–Schauder degree theory and obtained the existence of spatially nonhomogeneous solutions by regarding the diffusion coefficient d as a bifurcation parameter. Yi et al. [38] found that both spatially homogeneous and heterogeneous oscillatory solutions can be seen by using the composite form of some spatially independent parameters as a bifurcation parameter. As we known, nontrivial steady-state solutions and periodic