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The bijective soft set with its operations

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ABSTRACT

Soft set theory is a newly emerging tool to deal with uncertain problems and has been studied by scholars in theory and practice. This paper proposes the concept of bijective soft set and some of its operations such as the restricted *AND* and the relaxed *AND* operations on a bijective soft set, dependency between two bijective soft sets, bijective soft decision system, significance of bijective soft set with respect to bijective soft decision system, reduction of bijective soft set with respect to bijective soft decision system, and decision rules in bijective soft decision system. With these notions and operations, an application of bijective soft set in decision-making problems is also discussed.

1. Introduction

Complexities of modeling uncertain data in economics, engineering, environmental science, sociology, medical science, and many other fields are very important for solving practical problems. In [1], Molodtsov argued that classical methods are not always successful, because the uncertainties appearing in these domains may be of various types, and the reason for these difficulties is, possibly, the inadequacy of the parameterization tool of the theory. Consequently, he [1] initiated the concept of soft set theory. To avoid difficulties, soft set theory uses an adequate parameterization (see Definition 2.1). With the adequate parameterization of soft set theory, Zadeh's fuzzy set may be considered as a special case of the soft set [1]. On the issues of representation, Pei and Miao [2] represented the rough set model as two soft sets, Aktaş and Çagman [3] have proved that every rough set may be considered as a soft set, and write rough sets as predicates. Herawan and Deris [4] devoted to revealing interconnection between rough sets and soft sets and they presented a direct proof that Pawlak's and Iwinski's rough sets can be considered as soft sets. Moreover, Feng et al. [5] initiated concepts of soft–rough fuzzy sets, rough–soft sets, soft–rough sets, soft–rough fuzzy sets.

Recently, soft set theory has been developed rapidly and focused by some scholars in theory and practice. Based on the work of Molodtsov, Maji et al. [6] defined equality of two soft sets, subset and superset of soft set, complement of a soft set, null soft set, and absolute soft set with examples. They also defined soft binary operations such as *AND*, *OR* and the operation of union, intersection and De Morgan's law. Aktaş and Çagman [3] introduced the basic properties of soft sets to the related concept of fuzzy sets as well as rough sets, and then they gave a definition of soft group and derived the basic properties by using Molodtsov's definition of the soft sets. Liu and Yan [7] discussed the algebraic structure of fuzzy soft sets and gave the definition of fuzzy soft group. In their paper, they defined operations on fuzzy soft groups and proved some results on them as well; they also presented fuzzy normal soft subgroups and fuzzy soft homomorphism and discussed their properties. Jun and Park [8] proposed the notions of soft ideals and idealistic soft BCK/BCI-algebras, and gave several examples. They also provided the relations between soft BCK/BCI-algebras and idealistic soft BCK/BCI-algebras and established intersection, union, *AND* operation, and *OR* operation of soft ideals and idealistic soft BCK/BCI-algebras. Feng and Jun [9] introduced

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Table 2.1
The tabular representation of (F, E).

U	e_1	e_2	e_3	e_4
h_1	1	1	1	0
h_2	0	1	0	1
h_3	1	0	1	1
h_4	1	0	0	1

the notions of soft semirings, soft subsemirings, soft ideals, idealistic soft semirings and soft semiring homomorphisms. Ali et al. [10] corrected some mistakes of former studies and proposed some new operations on soft sets.

The applications of soft set theory are also extended to data analysis under incomplete information [11], combined forecasts [12], decision-making problems [13], normal parameter reduction [14], and d-algebras [15], demand analysis [16]. These applications showed the promising of soft set theory in handling uncertain problems.

This paper proposes a new type of soft set, bijective soft set. For the notion of Bijective soft set, every element can be only mapped into one parameter and the union of partition by parameter set is universe. Based on the notion of bijective soft set, we propose some of its operations to study the relationship between bijective soft sets. This paper formulates the notion of bijective soft set by the following steps. First, this paper proposes the concept of bijective soft set. Second, this paper proposes the restricted *AND* and the relaxed *AND* operation on a bijective soft set and boundary region. Third, this paper proposes the dependency between two bijective soft sets, and defines soft decision system on bijective soft sets, the reduction of bijective soft set with respect to bijective soft decision system, and studies the significance of bijective soft set in bijective soft decision system. Finally, this paper gives an application of bijective soft set in decision-making problems.

The rest of the paper is organized as follows. Section 2 introduces the basic principles of soft sets. Section 3 gives the concepts of bijective soft set, and some of its operations. Section 4 gives an application of bijective soft set in decision-making problems. Finally Section 5 presents some conclusions from the research.

2. Preliminaries

2.1. Soft sets

Let U be a common universe and let E be a set of parameters.

Definition 2.1 (*See [1]*)**.** A pair (F, E) is called a soft set (over U) if and only if F is a mapping of E into the set of all subsets of the set U, where F is a mapping given by $F : E \to P(U)$.

In other words, the soft set is a parameterized family of subsets of the set U. Every set $F(\varepsilon)$ $(\varepsilon \in E)$, from this family may be considered as the set of ε-elements of the soft sets (F, E), or as the set of ε-approximate elements of the soft set.

To illustrate this idea, let us consider the following example.

Example 1. Let universe $U = \{h_1, h_2, h_3, h_4\}$ be a set of houses, a set of parameters $E = \{e_1, e_2, e_3, e_4\}$ be a set of status of houses which stand for the parameters "beautiful", "cheap", "in green surroundings", and "in good location" respectively. Consider the mapping F be a mapping of E into the set of all subsets of the set U. Now consider a soft set (F, E) that describes the "attractiveness of houses for purchase". According to the data collected, the soft set (F, E) is given by

$$(F, E) = \{(e_1, \{h_1, h_3, h_4\}), (e_2, \{h_1, h_2\}), (e_3, \{h_1, h_3\}), (e_4, \{h_2, h_3, h_4\})\},$$

where $F(e_1) = \{h_1, h_3, h_4\}$, $F(e_2) = \{h_1, h_2\}$, $F(e_3) = \{h_1, h_3\}$ and $F(e_4) = \{h_2, h_3, h_4\}$. In order to store a soft set in computer, a two-dimensional table is used to represent the soft set (F, E). Table 2.1 is the tabular form of the soft set (F, E). If $h_i \in F(e_j)$, then $h_{ij} = 1$, otherwise $h_{ij} = 0$, where h_{ij} are the entries (see Table 2.1).

Definition 2.2 (*See [6]*)**.** For two soft sets (F, A) and (G, B) over U, (F, A) is called a soft subset of (G, B) if

(1) $A \subset B$ and
(2) $\forall \varepsilon \in A$, $F(\varepsilon)$ and $G(\varepsilon)$ are identical approximations.

This relationship is denoted by $(F, A)\tilde{\subset}(G, B)$.

Similarly, (F, A) is called a soft superset of (G, B) if (G, B) is a soft subset of (F, A). This relationship is denoted by $(F, A)\tilde{\supset}(G, B)$.

Example 2 (*See [6]*)**.** Let $A = \{e_1, e_3, e_5\} \subset E$ and $B = \{e_1, e_2, e_3, e_5\} \subset E$.

Clearly, $A \subset B$.

Let (F, A) and (G, B) be two soft set over the same universe $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ such that

$$G(e_1) = \{h_2, h_4\}, G(e_2) = \{h_1, h_3\}, G(e_3) = \{h_3, h_4, h_5\}, G(e_5) = \{h_1\}$$
$$F(e_1) = \{h_2, h_4\}, F(e_3) = \{h_3, h_4, h_5\}, F(e_5) = \{h_1\}.$$

Therefore, $(F, A)\tilde{\subset}(G, B)$.