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Augmented Lagrangian method for total generalized variation based Poissonian image restoration☆

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ABSTRACT

Instead of adopting the traditional total variation as a regularizer, this article introduces a second-order total generalized variation regularization scheme for deconvolving Poissonian image. Numerically, an efficient augmented Lagrangian method associated with alternating minimization method is described to obtain the optimal solution recursively. In addition, we provide the rigorous convergence analysis for the resulting algorithm at great length. Finally, compared with the total variation based efficient strategies, numerical simulations definitely indicate the competitive performance of our proposed approach to deblurring poissonian image, both in terms of restoration accuracy and edge-preserving ability.

1. Introduction

Removing Poisson noise is an important and challenging task in image processing and computer vision, such as astronomical imaging [1], medical imaging [2], electronic microscopy [3,4], as well as positron emission tomography [5].

One successful example of variational models for Poissonian image reconstruction is the one proposed by Le, Chartrand, and Asaki [6] using the total variation (TV) norm as regularization term, formulated as

$$\min_u \int_\Omega |\nabla u| + \lambda \int_\Omega (u - f \log u) \mathrm{d}x, \tag{1}$$

where $\Omega \subset \mathbb{R}^2$ is a bounded open subset with Lipschitz boundary, $\lambda > 0$ is a tuning parameter, f stands for the positive observed image, and u the unknown true data.

Considering the blurring effect, authors in [7–15] studied the TV regularization and Kullback–Leibler (KL) divergence fidelity based penalized likelihood approach to restoring Poissonian image by

$$\min_u \int_\Omega |\nabla u| + \lambda \int_\Omega (Ku - f \log Ku) \mathrm{d}x, \tag{2}$$

with K being a nonnegative linear compact operator, subject to $K \cdot 1 \neq 0$. To deal with the resulting convex optimization problem (2), there exist several efficient numerical algorithms. For instance, as shown in [7,8], authors proposed the classical expectation maximization (EM)–TV algorithm. Associated with the Bregman iteration and inverse scale space methods, Brune et al. [9] introduced an improved EM–TV algorithm. Subsequently, compared with the category of EM–TV based

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schemes, authors in [10–12] described a more efficient alternating split Bregman iteration. And furthermore, other efficient algorithms, such as nested iterative algorithm [13], scaled gradient projection method [14], alternating extra gradient method [15], as well as alternating direction algorithm [16,17] have been investigated in depth for TV regularizer based Poissonian images deconvolution recently.

As is well known, the numerous blocky effects are engendered in the recovered results owing to the TV regularizer framework. Aiming at conquering this drawback, many techniques, such as fourth-order partial differential equation (PDE) filter [18–21], hybrid regularizer model combining second-order and fourth-order PDEs [22,23], and nonlocal TV regularized scheme [24,25] have been researched widely and made great successes in image processing.

Recently, another improvement to avoid the staircase effect is the total generalized variation of order n with weight α (TGV_α^n) scheme, which was initially introduced as penalty functional for image restoration by Bredies, Kunisch and Pock in [26]. It is noteworthy that, as a generalization, TGV shares some favorable properties of TV, such as rotational invariance, lower semi-continuity and convexity. Differently, TGV involves and balances higher-order derivatives of u. This leads to that the recovered image using TGV regularization includes sharp edges and exempts from the staircase effect. Thereinto, equipping with the simplicity and eminent properties, TGV_α^2 regularized models have been widely researched recently, and achieved great successes in image restoration [27–30], image zooming [31] and medical imaging [32].

In the current article, we are interested in the TGV_α^2 regularization based model for deblurring Poissonian image. Mathematically, the optimization problem can be formulated as

$$\min_u \mathrm{TGV}_\alpha^2(u) + \lambda \int_\Omega (Ku - f \log Ku)\mathrm{d}x, \tag{3}$$

where $\Omega \subset \mathbb{R}^d$ denotes a bounded Lipschitz domain with the image dimension $d \geq 1$.

Our most important contribution is introducing a highly efficient alternating augmented Lagrangian method in detail, containing three auxiliary variables and tailed for solving the objective function (3) (i.e., TGV-KL model). Arguably, another important advantage is that the modified optimization scenario incorporates the TGV norm, such that the recovered image possesses sharp edges and avoids staircasing artifacts. As a consequence, this contributes to obtain more accurate and stable numerical solutions. Furthermore, provided experimental results demonstrate the high efficiency of our algorithm for removing Poisson noise, in comparison with several state-of-the-art methods.

This paper is structured as follows. In Section 2, we briefly describe the necessary definitions and notations on the proposed model. Section 3 is intended to establish the detailed augmented Lagrangian algorithm for solving the optimization problem. And the rigorous convergence proof of the advanced strategy is exhibited in Section 4. Finally, numerical experiments aiming at illustrating the outstanding performance of our methodology are provided in Section 5, while conclusions of this work are summarized in the last section.

2. Preliminaries

For latter reference, our objective in this section is to display the basic concepts and properties on the model (3). Thanks to the existing work shown in [26,27,33], we have

Definition 1. Let $\Omega \subset \mathbb{R}^d$ be a bound domain, $n \geq 1$ and $\alpha = (\alpha_0, \ldots, \alpha_{n-1}) > 0$. Then the total generalized variation of order n with weight α for $u \in L^1(\Omega)$ is defined as the value of the functional

$$\mathrm{TGV}_\alpha^n(u) = \sup\left\{\int_\Omega u\mathrm{div}^n\vartheta\,\mathrm{d}x \Big| \vartheta \in C_c^n(\Omega, \mathrm{Sym}^n(\mathbb{R}^d)), \|\mathrm{div}^l\vartheta\|_\infty \leq \alpha_l, l = 0, \ldots, n-1\right\}, \tag{4}$$

where $\mathrm{Sym}^n(\mathbb{R}^d)$ is the space of symmetric tensors of order n on $\mathbb{R}^d$, and $C_c^n(\Omega, \mathrm{Sym}^n(\mathbb{R}^d))$ the space of compactly supported symmetric tensor fields. An important remark is that, the space of bound generalized variation (BGV) equipped with

$$\mathrm{BGV}^n(\Omega) = \left\{u \in L^1(\Omega)\Big|\mathrm{TGV}_\alpha^n(u) < \infty\right\}, \quad \|u\|_{\mathrm{BGV}^n} = \|u\|_1 + \mathrm{TGV}_\alpha^n(u), \tag{5}$$

is also a Banach space for some weight $\alpha > 0$.

In this paper, we focus on the second-order TGV, which can be formulated as

$$\mathrm{TGV}_\alpha^2(u) = \sup\left\{\int_\Omega u\mathrm{div}^2\vartheta\,\mathrm{d}x\Big|\vartheta \in C_c^2(\Omega, S^{d\times d}), \|\vartheta\|_\infty \leq \alpha_0, \|\mathrm{div}\vartheta\|_\infty \leq \alpha_1\right\}, \tag{6}$$

where $S^{d\times d}$ is the set of all symmetric $d \times d$ matrices, $(\mathrm{div}\vartheta)_i = \sum_{j=1}^d \frac{\partial \vartheta_{ij}}{\partial x_j}$, $\mathrm{div}^2\vartheta = \sum_{i,j=1}^d \frac{\partial^2\vartheta_{ij}}{\partial x_i \partial x_j}$, and the infinity norm of ϑ and $\mathrm{div}\vartheta$ are given by

$$\|\vartheta\|_\infty = \sup_{x\in\Omega}\Big(\sum_{i,j=1}^d |\vartheta_{ij}(x)|^2\Big)^{1/2}, \qquad \|\mathrm{div}\vartheta\|_\infty = \sup_{x\in\Omega}\Big(\sum_{i=1}^d |(\mathrm{div}\vartheta)_i(x)|^2\Big)^{1/2}.$$