Computers and Mathematics with Applications 71 (2016) 991–1009

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Decay of solutions for a mixture of thermoelastic solids with different temperatures

Jaime E. Muñoz Rivera [a,b], Maria Grazia Naso [c,*], Ramon Quintanilla [d]

[a] National Laboratory for Scientific Computation, Rua Getulio Vargas 333, Quitadinha-Petrópolis 25651-070, Rio de Janeiro, RJ, Brazil
[b] Instituto de Matemática - UFRJ, Av. Horácio Macedo, Centro de Tecnologia, Cidade Universitaria - Ilha do Fundão 21941-972, Rio de Janeiro, RJ, Brazil
[c] DICATAM, Università degli Studi di Brescia, Via Valotti 9, 25133 Brescia, Italy
[d] Departament de Matemàtiques, UPC C. Colón 11, 08222 Terrassa, Barcelona, Spain

ARTICLE INFO

Article history:
Received 21 March 2015
Received in revised form 5 January 2016
Accepted 12 January 2016
Available online 6 February 2016

Keywords:
Thermoelastic mixtures
Exponential decay
Weakly coupled system

ABSTRACT

We study a system modeling thermomechanical deformations for mixtures of thermoelastic solids with two different temperatures, that is, when each component of the mixture has its own temperature. In particular, we investigate the asymptotic behavior of the related solutions. We prove the exponential stability of solutions for a generic class of materials. In case of the coupling matrix **B** being singular, we find that in general the corresponding semigroup is not exponentially stable. In this case we obtain that the corresponding solution decays polynomially as $t^{-1/2}$ in case of Neumann boundary condition. Additionally, we show that the rate of decay is optimal. For Dirichlet boundary condition, we prove that the rate of decay is $t^{-1/6}$. Finally, we demonstrate the impossibility of time-localization of solutions in case that two coefficients (related with the thermal conductivity constants) agree.

1. Introduction

Under the theory of non-classical elastic solids we understand certain generalizations of the classical theory of elasticity. The most known non-classical elastic solids are the elastic solids with voids, micropolar elastic solids, nonsimple elastic solids and the mixtures of elastic solids. Micropolar elastic solids have first been introduced by the Cosserat brothers at the beginning of the last century and they were revisited, analyzed and extended by Eringen and many other researchers in the second part of the past century. For an overview on these so called microcontinuum theories we refer, e.g., to [1–3]. In the same period, the theories concerning the nonsimple materials, materials with voids and mixtures of material were established. It is worth recalling here the book of Ieşan [4] where several of these theories are analyzed. This manuscript is concerned with one of these theories: the mixtures of elastic solids.

Thermoelastic mixtures of solids have deserved a big interest in the last decades (see, e.g., [5–12]). Qualitative properties of solutions to the problems defining this kind of materials have been the scope of many investigations. Several results concerning existence, uniqueness, continuous dependence and asymptotic stability can be found in the literature [13–19]. In this paper, we study the decay of solutions in case of a one-dimensional rod composed by a mixture of two thermoelastic solids with two different temperatures. We will prove the exponential stability in a generic case, however, we cannot expect

* Corresponding author.
E-mail addresses: rivera@lncc.br (J.E. Muñoz Rivera), mariagrazia.naso@unibs.it (M.G. Naso), ramon.quintanilla@upc.edu (R. Quintanilla).

http://dx.doi.org/10.1016/j.camwa.2016.01.010

that the solutions can identically vanish after a finite time and we will see the impossibility of localization for various scenarios. In several situations, the decay is not so fast and we will prove the polynomial decay for these situations. It is worth recalling that studying the rate of decay of the solutions for several non-classical theories has been the goal of many articles in this last decade [20–23]. Thus, the present paper aims to be a new contribution in this line.

For a rod composed by a mixture of two interacting continua occupying the interval $(0,\ell)$ the displacements of each component of typical particles at time t are denoted, by u and w, respectively, where $u=u(x,t):(0,\ell)\times(0,T)\to\mathbb{R}$ and $w=w(y,t):(0,\ell)\times(0,T)\to\mathbb{R}$, with $T>0$. We assume that the particles under consideration are in the same position at time $t=0$, so that $x=y$.

We also assume the existences of two different temperatures (see [24]), in each point x and at time t, given by $\theta_i=\theta_i(x,t):(0,\ell)\times(0,T)\to\mathbb{R},\ i=1,2$. We denote by $\rho_i,\ i=1,2$ the mass density of each constituent at time $t=0$. We introduce $\mathcal{T}$ and $\mathcal{S}$ as the partial stresses associated with these two constituents, P the internal diffusive force, $\Xi^{(i)},\ i=1,2,$ the entropy densities, $Q^{(i)},\ i=1,2,$ the heat flux vector and $\mathfrak{T}_0$ is the absolute temperature in the reference configuration. In the absence of body forces, the system consists of the following equations:

– equations of motion

$$\rho_1 u_{tt}=\mathcal{T}_x-P,\qquad \rho_2 w_{tt}=\mathcal{S}_x+P, \tag{1.1}$$

– energy equations

$$\rho_1\mathfrak{T}_0\,\Xi^{(1)}{}_t=Q_x^{(1)}+W^{(1)}+G,\qquad \rho_2\mathfrak{T}_0\,\Xi^{(2)}{}_t=Q_x^{(2)}+W^{(2)}-G, \tag{1.2}$$

– constitutive equations

$$\mathcal{T}=a_{11}u_x+a_{12}w_x-\beta_1\theta_1-\beta_2\theta_2,\qquad \mathcal{S}=a_{12}u_x+a_{22}w_x-\gamma_1\theta_1-\gamma_2\theta_2, \tag{1.3}$$

$$P=\alpha(u-w),\qquad G=-a(\theta_1-\theta_2), \tag{1.4}$$

$$\rho_1\,\Xi^{(1)}=\beta_1u_x+\beta_2w_x+M_1^{(1)}\theta_1+M_2^{(1)}\theta_2+T_0^{-1}\rho_2\kappa_1^{(2)}(\theta_1-\theta_2), \tag{1.5}$$

$$\rho_2\,\Xi^{(2)}=\gamma_1u_x+\gamma_2w_x+M_2^{(1)}\theta_1+M_2^{(2)}\theta_2+T_0^{-1}\rho_1\kappa_2^{(1)}(\theta_1-\theta_2), \tag{1.6}$$

$$Q^{(1)}=K_{11}\theta_{1,x}+K_{12}\theta_{2,x},\qquad Q^{(2)}=K_{21}\theta_{1,x}+K_{22}\theta_{2,x}. \tag{1.7}$$

Functions $W^{(i)}$ are given by

$$W^{(1)}=\rho_1\kappa_2^{(1)}\theta_{2,t}-\rho_2\kappa_1^{(2)}\theta_{1,t},\qquad W^{(2)}=\rho_2\kappa_1^{(2)}\theta_{1,t}-\rho_1\kappa_2^{(1)}\theta_{2,t}. \tag{1.8}$$

If we denote

$$b_1=T_0M_1^{(1)}+2\rho_2\kappa_1^{(2)},\qquad b_2=T_0M_2^{(1)}-\rho_1\kappa_2^{(1)}-\rho_2\kappa_1^{(2)},\qquad b_3=M_2^{(2)}+2\rho_2\kappa_2^{(1)},$$

and substitute constitutive equations (1.3)–(1.8) into dynamical equations (1.1)–(1.2), we obtain the following evolution system

$$\begin{array}{ll}
\rho_1u_{tt}-a_{11}u_{xx}-a_{12}w_{xx}+\alpha(u-w)+\beta_1\theta_{1,x}+\beta_2\theta_{2,x}=0 & \text{in } (0,\ell)\times(0,T),\\
\rho_2w_{tt}-a_{12}u_{xx}-a_{22}w_{xx}-\alpha(u-w)+\gamma_1\theta_{1,x}+\gamma_2\theta_{2,x}=0 & \text{in } (0,\ell)\times(0,T),\\
b_1\theta_{1,t}+b_2\theta_{2,t}-K_{11}\theta_{1,xx}-K_{12}\theta_{2,xx}+\beta_1u_{xt}+\beta_2w_{xt}+a(\theta_1-\theta_2)=0 & \text{in } (0,\ell)\times(0,T),\\
b_2\theta_{1,t}+b_3\theta_{2,t}-K_{21}\theta_{1,xx}-K_{22}\theta_{2,xx}+\gamma_1u_{xt}+\gamma_2w_{xt}-a(\theta_1-\theta_2)=0 & \text{in } (0,\ell)\times(0,T),
\end{array}$$

where, for the sake of simplicity, we assume that $\mathfrak{T}_0=1$. Therefore the corresponding evolution system can be written as

$$\mathbf{R}_1U_{tt}-\mathbf{A}U_{xx}+\alpha\mathbf{N}U+\mathbf{B}\Upsilon_x=0,\quad \text{in } (0,\ell)\times(0,T), \tag{1.9}$$

$$\mathbf{R}_2\Upsilon_t-\mathbf{K}\Upsilon_{xx}+a\mathbf{N}\Upsilon+\mathbf{B}U_{xt}=0,\quad \text{in } (0,\ell)\times(0,T). \tag{1.10}$$

Here

$$U=\begin{pmatrix}u\\ w\end{pmatrix},\qquad \Upsilon=\begin{pmatrix}\theta_1\\ \theta_2\end{pmatrix},$$

$$\mathbf{A}=(a_{ij})_{2\times2},\qquad \mathbf{R}_1=(\delta_{ij}\rho_i)_{2\times2},\qquad \mathbf{R}_2=(b_i)_{2\times2},$$

are symmetric matrices, δ_{ij} is the usual Kronecker's delta, $\mathbf{K}=(K_{ij})_{2\times2}$, $\mathbf{N}=((-1)^{i+j})_{2\times2}$ and $\mathbf{B}=\begin{pmatrix}\beta_1&\beta_2\\ \gamma_1&\gamma_2\end{pmatrix}\in\mathbb{R}^{2\times2}$. In general $\mathbf{B}$ is neither symmetrical nor positive definite. We supplement our system with the initial conditions

$$U(x,0)=\begin{pmatrix}u_0\\ w_0\end{pmatrix},\qquad U_t(x,0)=\begin{pmatrix}u_1\\ w_1\end{pmatrix},\qquad \Upsilon(x,0)=\begin{pmatrix}\theta_{10}\\ \theta_{20}\end{pmatrix}, \tag{1.11}$$

and boundary conditions

$$U(0,t)=U(\ell,t)=\Upsilon(0,t)=\Upsilon(\ell,t)=0. \tag{1.12}$$