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A second order dispersive FDTD algorithm for transverse electric Maxwell's equations with complex interfaces

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ABSTRACT

This work overcomes the difficulty of the finite-difference time-domain (FDTD) algorithm in solving the transverse electric (TE) Maxwell's equations with inhomogeneous dispersive media. For such TE problems, the electric fields are discontinuous across the dispersive interfaces. Moreover, such discontinuities are time variant. A novel matched interface and boundary time-domain (MIBTD) method is proposed to solve such problems through new developments in both mathematical formulations and numerical algorithms. Mathematically, instead of handling all zeroth and first order jump conditions in a local coordinate, we directly construct the TE jump conditions which are needed in the FDTD computations in the Cartesian coordinate. Such Cartesian direction conditions depend on the time, as well as tangential and normal components of the electric flux. Driven by the jump condition modeling, we adopt the standard Maxwell's equations in coupling with the Debye auxiliary differential equations for the electric flux as the governing equations. Computationally, the leapfrog scheme is employed for integrating the Maxwell system and time dependent jump conditions. Sophisticated interface treatments are developed in both producing the TE jump conditions and enforcing them in the FDTD algorithm, based on a staggered Yee lattice. The numerical accuracy, stability, and efficiency of the proposed scheme are investigated by considering dispersive interfaces of various shapes. The MIBTD method achieves a spatially second order of convergence in all tests. To the best of our knowledge, the present MIBTD scheme is the first FDTD method in the literature that can restore second order accuracy in treating curved dispersive interfaces for the TE Maxwell system.

1. Introduction

It is well known that the finite-difference time-domain (FDTD) algorithm introduced by Yee [1] has been a main workhorse for solving Maxwell's equations in various applications of computational electromagnetics (CEM) [2]. Based on the Yee lattice — a simple staggered Cartesian grid, the FDTD method is very simple and efficient. However, in solving CEM problems with inhomogeneous media and complex geometries, the staircase representation of curved material interfaces usually undermines the Yee scheme so that its accuracy is reduced from second order to first order [3].

This motivates the development of various embedded FDTD methods for solving Maxwell's equations [4–10,3,11]. In these methods, sophisticated interface treatments are conducted near a curved interface by imposing the jump conditions in the FDTD discretizations, while away from the interface, the standard FDTD approximation is performed. The embedded

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FDTD methods not only maintain the simplicity and computational efficiency of the Yee scheme, but also can restore second order accuracy in treating curved dielectric interfaces based on a simple Cartesian grid. We note that for the dispersive interface problems to be considered in this work, the dispersive counterparts of the embedded FDTD methods are much less well studied in the CEM literature.

In the present context, a dispersive interface is defined to be a material boundary separating two inhomogeneous media, among which at least one is a dispersive medium. Dispersive material, in which the permittivity is a function of frequency, is widely presented in nature, such as in soil, rock, ice, snow, and biological tissues. Thus, the material dispersiveness has to be considered in many CEM applications, such as microwave imaging for early detection of breast cancer [12] and ground penetrating radar [13]. Various numerical approaches have been successfully developed in the literature to solve Maxwell's equations in dispersive medium, including finite element time domain methods [14–16], discontinuous Galerkin time domain methods [17–19], and dispersive FDTD methods [20–23]. In dealing with inhomogeneous media, smoothing techniques or effective permittivity approaches have been considered in [24–26] to reduce the staircase approximation errors of the dispersive FDTD methods.

Nevertheless, there exist great difficulties that hinder the generalization of the high order embedded FDTD methods [4–10,3,11] for solving dispersive interface problems. Across a non-dispersive or dielectric interface, the Maxwell wave solution is known to be non-smooth or even discontinuous [3,11], while the jumps in the wave solution and its derivatives are time independent and can be analytically described through the jump conditions. The success of the embedded FDTD methods actually lies in their effective enforcement of the jump conditions in the FDTD discretization. However, because a broadband electromagnetic wave is propagated in a frequency dependent manner in a dispersive medium, the wave solution will lose its regularity in a time variant manner across a dispersive interface [27]. In other words, the jump conditions in dispersive interface problems are time dependent. The extension of the existing embedded FDTD methods for modeling the time dependent jump conditions are highly nontrivial.

Recently, we have developed a family of embedded FDTD methods based on the matched interface and boundary (MIB) technique [11,28] for solving dielectric interface problems [11,29,30] and dispersive interface problems [27,31–33]. In particular, for dispersive MIB time-domain (MIBTD) methods, we have examined transverse magnetic (TM) Maxwell's equations with both Debye [27,31,32] and Drude [33] dispersive models. The popular auxiliary differential equation (ADE) approach is utilized to incorporate the dispersive models in the time domain computation. Novel hybrid Maxwell formulations have been proposed in these works [27,31–33] by coupling the second order wave equation for the electric component with Maxwell's equations for the magnetic components. Consequently, one can track the time dependent parts of the jump conditions across a dispersive interface. Through the introduction of fictitious values and an iterative use of zeroth and first order jump conditions [11,28], the MIB scheme is employed to impose the time dependent TM jump conditions over a staggered Yee lattice. The MIB modeling is systematically carried out so that orders up to 6 have been achieved numerically for solving straight dispersive interfaces [27,31]. For curved dispersive interfaces, the MIBTD methods [32,33] are the only known embedded FDTD algorithms that secure a second order of accuracy, to the best of the authors' knowledge.

However, in the existing dispersive MIBTD methods [27,31–33], only TM Maxwell's equations with continuous electric and magnetic field components are considered. These MIBTD methods cannot be directly generalized to solve transverse electric (TE) Maxwell's equations, which now have discontinuous wave solutions across the dispersive interface. In particular, the essential challenge of the MIBTD formulation for TE systems is due to the zeroth order jump conditions for the electric field $\vec{E} = (E_x, E_y, E_z)^T$. For the TM mode, we have simply one jump condition saying that E_z is continuous [27,31,32], whereas in the TE mode, both E_x and E_y are discontinuous across the dispersive interface. Moreover, the jumps in E_x and E_y will be time dependent and are coupled with the unknown electric flux density components D_x and D_y. Therefore, the construction of new TE jump conditions within a proper Maxwell formulation is indispensable for developing a embedded FDTD method for solving TE type dispersive interface problems.

The objective of this paper is to develop a novel second order MIBTD method for solving the TE Maxwell's equations with smooth curved interfaces and discontinuous wave solutions. The single-order Debye dispersion model [34] will be considered, whereas the proposed scheme can be similarly constructed for other dispersive media. In our previous TM studies [27,31–33], we usually first derive all zeroth order and first order jump conditions for all involved electromagnetic variables along both normal and tangential directions. Then rotate these jump conditions to the Cartesian directions. We will not attempt to do so for H_z, E_x and E_y in the present work, because this is too complicated in the TE case. To overcome this difficulty, we propose to directly construct TE jump conditions in Cartesian directions and focus only on conditions that are needed in correcting the FDTD discretization of the TE Maxwell's equations. Similar to our previous TM studies [27,31–33], the ADE approach is employed to couple the Debye dispersion model with the TE equations so that interface auxiliary different equations (IADEs) can be derived for describing the regularity changes in the proposed TE jump conditions. Another major difference from our previous TM studies [27,31–33] is that we will not use the hybrid Maxwell governing equations. Instead, the standard TE Maxwell's equations will be adopted, because they turn out to be more convenient for the current modeling of the TE jump conditions. Because of this, the leapfrog scheme is utilized in the present study for time integration. This is more efficient than the fourth order Runge–Kutta scheme used in our previous TM works [27,31–33] for solving the hybrid Maxwell's equations. Finally, the spatial approximations underlying the time dependent TE jump conditions are more complicated than those for the TM mode, because the proposed TE conditions couple normal and tangential information with the Cartesian jumps. Sophisticated extrapolation and interpolation schemes in both horizontal and vertical directions are developed to address this difficulty. With these theoretical and numerical developments, the