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Stock loan valuation under a stochastic interest rate model

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ABSTRACT

Stock loans are loans collateralized by stocks. They are modern financial products designed for investors with large equity positions. Mathematically, stock loans can be regarded as American call options with a time-dependent strike price. This study is the first in the literature that considers the valuation of stock loans in a stochastic interest rate framework. Based on portfolio analysis, a partial differential equation (PDE) governing the value of stock loans is derived. A set of appropriate boundary conditions, particularly in the interest rate direction, are also proposed to close the pricing system. A sound justification is provided for the proposed boundary conditions mathematically as well as financially. To solve the proposed nonlinear PDE system, a predictor–corrector finite difference method is adopted. Moreover, an alternating direction implicit (ADI) method is used to improve the computational efficiency. Numerical results suggest that the current method is reliable and the stochastic interest rate leads to a higher optimal exercise price of the stock loan in comparison with that calculated from the Black–Scholes model.
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1. Introduction

Stock loans are loans using stocks as collateral. To establish a stock loan, an investor who owns stocks delivers his stocks to a financial institution that provides stock loan service. After being charged an amount as the service fee of the stock loan, the investor will be granted a principal and a right which allows him/her to redeem his/her stocks at any valid time by repaying the principal and the loan interest. In the literature, this right is also referred to as the value of a stock loan. Considering its early exercise nature and the payoff, pricing a stock loan is actually equivalent to pricing an American call option with a time-dependent strike price.

Stock loans can suit various demands of investors. For instance, stock loans are used by risk aversion investors to transfer the risk of holding stocks to financial institutions. For stock holders who need cash but face selling restrictions, stock loans can overcome the barrier and establish market liquidity [1].

Stock loans have drawn increasing attentions in the academic world since 2007. The pricing of stock loans was first studied by Xia and Zhou under the Black–Scholes framework [1]. They assumed that the stock loan has an infinite expiration date and derived closed-form analytical expression for its price. Liang et al. [2,3] added automatic termination clause, cap and margin into the valuation of stock loans and formulated the pricing as a variational inequality. For stock loans with finite maturity, closed-form analytical solution has not yet been achieved, and most of the work are done approximately. For example, Dai and Xu [4] analysed stock loans in four different dividend distribution cases and calculated the prices using the binomial tree method. Lu and Putri [5] also considered the pricing of stock loans in the four divided distribution cases by using the Laplace transform method. For the valuation of stock loans under more complicated models, Pascucci et al. [6]

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formulated the pricing of stock loans with accumulative dividends as an obstacle problem. They analysed the properties of the solution and solved the price of the stock loans using the Lagrange finite element method. Prager and Zhang [7] derived closed-form analytical formulae for European-style stock loans with finite maturities under the classical geometric Brownian motion, the mean-reverting model and the two-state regime-switching model. Wong and Wong [8] studied this pricing problem under a fast mean-reverting stochastic volatility model by means of the asymptotic expansion method. Grasselli and Gomez [9] further investigated stock loans in an incomplete market and discussed the impacts of model parameters based on the numerical results computed by the projected successive-over-relaxation (PSOR) method.

In all the work mentioned above, the risk-free interest rate is assumed to be a constant. Market observations, however, suggest that the constant interest rate cannot provide a description of how the interest rate evolves with respect to the time [10], especially for financial products that have long time horizon, such as stock loans whose time horizon could expand over 20 or even 30 years. Since the interest rate has an important impact on the redeem strategy of stock loans, it is of both practical and theoretical interests to consider the pricing of stock loans under a stochastic interest rate framework. This is also the first time that the valuation of stock loans is considered under such a framework.

In this paper, we consider the pricing of stock loans with stochastic interest rate. By portfolio analysis [11], a two-dimensional Partial Differential Equation (PDE) governing the price of stock loans is obtained for the first time. Appropriate boundary conditions are then prescribed to close the PDE system. In particular, the boundary conditions along the interest rate direction are well discussed both mathematically and financially. Before the numerical method is introduced, several transformations are made to simplify the pricing system. A hybrid finite difference approach based on the predictor–corrector framework is then applied to solve the PDE system numerically. For efficiency, the ADI method is adopted in the correction phase.

In comparison to various numerical approaches for solving two-dimensional parabolic equations, the current method has the following unique features. Firstly, our method requires almost the same storage space as a one-dimensional problem and it will not increase even when this method is applied to multi-asset problems. Secondly, in addition to the option values, the present method can produce the entire optimal exercise boundary as part of the solution procedure, whereas in most of the methods, the optimal exercise prices cannot be obtained simultaneously, and need to be solved with some additional effort. Finally, our method requires no iterations, and can be easily extended to solve other nonlinear problems.

The rest of the paper is organized as follows. In Section 2, we provide the derivation of the governing equation and discussions of boundary conditions. Transformations and the predictor–corrector approach are illustrated in Section 3. In Section 4, numerical examples are presented together with some discussions. Concluding remarks are given in Section 5.

2. Mathematical formulation

This section provides the mathematical formulation of stock loans. It is further divided into two subsections, according to two important issues to be addressed. The governing equation is derived in the first subsection whereas the boundary conditions are discussed in the second one.

2.1. Governing PDE for stock loans

As pointed out in [1], once a stock loan is established, it can be regarded as a client buying an American call option with a time-dependent strike price $Ke^{\gamma t}$ at a price $(S-K+c)$, where S is the stock price, K is the principal, c is the service fee charged by the financial institution, and γ is the continuously compounded loan interest rate. By the "no arbitrage assumption", we have $V = S - K + c$, where V is the value of the stock loan.

We now assume that both the stock price and the risk-free interest rate follow the Geometric Brownian motion in Itô's form as:

$$\begin{aligned} dS &= (r - D)Sdt + \sigma_1 SdW_1, \\ dr &= ardt + \sigma_2 rdW_2, \end{aligned} \tag{1}$$

where r is the risk-free interest rate, D is the continuously compounded dividend yield, and σ_1 and σ_2 are constant volatilities of the stock price and the risk-free interest rate, respectively. Since there is no drift term in the Dothan interest rate model [12], we further set $a = 0$. The terms W_1 and W_2 are two standard Brownian motions correlated with a factor ρ, i.e., $E[dW_1 dW_2] = \rho dt$. Financially, ρ should be restricted to $[-1, 0]$ because a higher interest rate usually yields a lower stock price [13].

In the following work, we shall focus on a common stock loan which has a finite life span and its dividend is paid to the financial institution until the redemption occurs and will never be returned to the investor. In fact, in the financial market, there are also perpetual stock loans which will never be expired. However, the pricing of these perpetual stock loans is totally different from the current case, and will be discussed in a forthcoming paper.

To derive the governing equation, we construct a portfolio consisting of a long position in Δ_1 shares of stock S, Δ_2 shares of default free bond P and a short position in one share of stock loan V. In particular, the bond P pays no coupon and satisfies the following PDE:

$$\frac{\partial P}{\partial t} + \frac{1}{2}\sigma_2^2 r^2 \frac{\partial^2 P}{\partial r^2} - \lambda\sigma_2 r\frac{\partial P}{\partial r} - rP = 0, \tag{2}$$

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