Computers and Mathematics with Applications 70 (2015) 1799–1816

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Multilevel finite element discretizations based on local defect correction for nonsymmetric eigenvalue problems ☆

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ARTICLE INFO

Article history:
Received 16 December 2014
Received in revised form 1 August 2015
Accepted 2 August 2015
Available online 21 August 2015

Keywords:
Nonsymmetric eigenvalue problems
Finite element
Multilevel discretization
Local refinement
Local defect correction

ABSTRACT

Based on the work of Xu and Zhou (2000), we establish new three-level and multilevel finite element discretizations by local defect correction techniques. Theoretical analysis and numerical experiments show that the discretizations are simple and easy to implement, and can be used to solve nonsymmetric eigenvalue problems with non smooth eigenfunctions efficiently. We also discuss the local error estimates of finite element approximations; it is a new feature here that the estimates apply to the local domains containing corner points.

1. Introduction

Nonsymmetric elliptic eigenvalue problems have an important physical background, such as convection–diffusion in fluid mechanics, environmental problems and so on. Thus finite element methods for solving nonsymmetric eigenvalue problems have become an important topic which has attracted the attention of mathematical and physical fields: [1] discussed a priori error estimates, [2–7] a posteriori error estimates and adaptive algorithms, [8] function value recovery algorithms, [9] two level algorithms, [10,11] extrapolation methods, [2] an adaptive homotopy approach, etc. This paper turns to discuss multilevel finite element discretizations based on local defect correction. The defect correction, also called the residual correction or the iterative improvement, is a technique in numerical linear algebra that can effectively improve the accuracy of solutions of linear algebra equations (e.g., see Section 2.5 in [12]). In 2000, Xu and Zhou [13] combined the defect correction with the finite element method to propose the local defect correction technique for the parallel-computing of elliptic equations. This technique has been developed by He et al. [14,15], Xu and Zhou [16], Dai and Zhou [17], Bi et al. [18] and Yang and Han [19], and successfully applied to Stokes equations, symmetric elliptic eigenvalue problems with non smooth eigenfunctions (including the electronic structure problems) and Stokes eigenvalue problem.

In this paper, we further apply the local defect correction technique to nonsymmetric elliptic eigenvalue problems with non smooth eigenfunctions and (or) convection dominated terms. Our work has the following features. (1) We extend local and parallel three-level finite element discretizations for symmetric eigenvalue problems established by Dai and Zhou [17] to nonsymmetric eigenvalue problems. (2) Based on the transition layer technique in [18], we establish new multilevel finite element discretizations with local refinement, this scheme repeatedly makes defect correction on finer and finer local meshes to make up for abrupt changes of the local mesh size caused by the three level scheme. As expected, theoretical

☆ Project supported by the National Natural Science Foundation of China (Grant NO. 11161012).
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http://dx.doi.org/10.1016/j.camwa.2015.08.010
0898-1221/

analysis and numerical experiments show that our schemes are simple and easy to implement, and can be used to solve nonsymmetric eigenvalue problems with non smooth eigenfunctions and (or) convection dominated terms. Numerical experiments show that, compared with the adaptive homotopy approach in [2], our algorithm is also efficient. (3) For the nonsymmetric problems, based on the work of [20,13], we discuss the local error estimates of finite element approximations; it is a new feature here that the estimates apply to the local domains containing corner points (see Theorem 2.1, Lemma 2.3 and Remark 2.1 in this paper).

In this paper, regarding the basic theory of finite elements, we refer to [21–24].

2. Preliminaries

Consider the nonsymmetric elliptic differential operator eigenvalue problem:

$$Lu \equiv -\sum_{i,j=1}^{d} \partial_j(a_{ij}(x)\partial_i u) + \sum_{i=1}^{d} b_i(x)\partial_i u + c(x)u = \lambda m(x)u, \quad \text{in } \Omega, \tag{2.1}$$

$$u = 0, \quad \text{on } \partial\Omega, \tag{2.2}$$

where $\Omega \subset R^d$, $d \geq 2$, is a polyhedral bounded domain with boundary $\partial\Omega$, $\partial_i u = \frac{\partial u}{\partial x_i}$, $i = 1, 2, \ldots, d$.

Let

$$a(u, v) = \int_{\Omega} \left(\sum_{i,j=1}^{d} a_{ij}\partial_i u \partial_j \overline{v} + \sum_{i=1}^{d} b_i \partial_i u \overline{v} + cu\overline{v} \right) \mathrm{d}x,$$

$$b(u, v) = \int_{\Omega} mu\overline{v}\mathrm{d}x.$$

The variational form associated with (2.1)–(2.2) is given by: find $\lambda \in \mathbb{C}$ and $u \in H_0^1(\Omega)$ with $\|u\|_0 = 1$ satisfying

$$a(u, v) = \lambda b(u, v), \quad \forall\, v \in H_0^1(\Omega). \tag{2.3}$$

Assume that $a_{i,j}, b_i \in W_{1,\infty}(\Omega)$, $c \in L_\infty(\Omega)$ are given real or complex functions in Ω, $m \in L_\infty(\Omega)$ is a given real function which is bounded below by a positive constant in Ω. L is assumed to be uniformly strongly elliptic in Ω, i.e., there is a positive constant a_0 such that

$$Re \sum_{i,j=1}^{d} a_{i,j}\xi_i\xi_j \geq a_0 \sum_{i=1}^{d} \xi_i^2, \quad \forall x \in \Omega,\ \forall (\xi_1, \xi_2, \ldots, \xi_d) \in R^d. \tag{2.4}$$

We assume without loss of generality that $Re\, c \geq a_0/2 + \max_{1\leq i\leq d; x\in\Omega} |b_i(x)|^2/(2a_0)$ since adding a multiple of $m(x)$ to $c(x)$ only shifts the eigenvalues. Under above assumptions, we have

$$Re\, a(u, u) \geq \frac{1}{2} a_0 \|u\|_1^2, \quad \forall u \in H^1(\Omega); \tag{2.5}$$

and there are constants M_1 and M_2 such that

$$|a(u, v)| \leq M_1 \|u\|_1 \|v\|_1, \quad \forall u, v \in H^1(\Omega), \tag{2.6}$$

$$|b(u, v)| \leq M_2 \|u\|_0 \|v\|_0, \quad \forall u, v \in L_2(\Omega). \tag{2.7}$$

For $D \subset \Omega_0 \subset \Omega$, we use $D \subset\subset \Omega_0$ to mean that $dist(\partial D \setminus \partial\Omega, \partial\Omega_0 \setminus \partial\Omega) > 0$.

Assume that $\pi_h(\Omega) = \{\tau\}$ is a mesh of Ω with the mesh size function $h(x)$ whose value is the diameter h_τ of the element τ containing x, and $h(\Omega) = \max_{x\in\Omega} h(x)$ is the mesh diameter of $\pi_h(\Omega)$. We write $h(\Omega)$ as h for simplicity. Let $V^h(\Omega) \subset C(\overline{\Omega})$, defined on $\pi_h(\Omega)$, be a piecewise polynomial space, and $V_0^h(\Omega) = \{v | v \in V^h(\Omega), v|_{\partial\Omega} = 0\}$. Given $G \subset \Omega$, we define $\pi_h(G)$ and $V^h(G)$ to be the restriction of $\pi_h(\Omega)$ and $V^h(\Omega)$ to G, respectively, and

$$V_0^h(G) = \{v | v \in V^h(G), v|_{\partial G} = 0\}, \qquad V_h^0(G) = \{v \in V_0^h(\Omega) : \mathrm{supp}\, v \subset\subset G\}.$$

For any $G \subset \Omega$ mentioned in this paper, we assume that it aligns with $\pi_h(\Omega)$ when necessary.

In this paper, C denotes a positive constant independent of h, which may not be the same constant in different places. For simplicity, we use the symbol $\alpha \lesssim \beta$ to mean that $\alpha \leq C\beta$.

We adopt the following assumptions for meshes and finite element spaces, where (A0)–(A2) can be found in [13] and (A3) is also based on [13].

(A0) There exists $\nu \geq 1$ such that $h(\Omega)^\nu \lesssim h(x)$, $\forall x \in \Omega$.

This is apparently a very mild assumption, and most practical meshes including locally refined meshes should satisfy it.