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Improving the mechanical performances of a multilayered plate with the orientations of its layers of fibers

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ABSTRACT

We consider a symmetric composite multilayered plate whose fiber orientation varies from a layer to another. The plate model used is that of Mindlin. We are interested in determining the optimal fiber orientations that optimize, in the same time, two criteria: minimizing the compliance and maximizing the smallest eigenfrequency of vibration or minimizing both of the compliance and the smallest eigenfrequency or maximizing both of the smallest eigenfrequency and the smallest buckling load. In order to optimize one of the above criteria, a metaheuristic algorithm of Simulated Annealing type is used. While, in the case of optimizing two objectives, the Pareto front method is used. Numerical results are presented for a rectangular eight-layered plate.

1. Introduction

We consider a multilayered composite plate which is symmetric with respect to its mid-plane, of thickness 2ε, and composed of $2m$ layers of fibers whose orientations vary from a layer to another, θ_i stands for the angle between fibers of layer i and the axis x_1, see Fig. 1. Every layer of fibers is assumed to be homogeneous and orthotropic with respect to its local coordinate system consisting of the fiber axis and its perpendicular. Hence, the multilayered plate is homogeneous but anisotropic. It is desirable that the multilayered composite plate should be very resistant and very light, but it should also be well suited to parametric optimization; the parameters are the fiber orientations, the kind of material and the thicknesses of the different layers, [1–12,29]. Indeed, in order to avoid the resonance of a structure under a given excitation, its eigenfrequencies have previously been controlled by its mass (location of some masses in adequate places on the plate). For a multilayered composite plate, we can also control the eigenfrequencies by its rigidity: find the optimal orientations of the layers of fibers, $\theta^* = (\theta_1^*, \theta_2^*, \dots, \theta_m^*) \in [0, \pi]^m$, for which the spectrum of eigenfrequencies does not intercept the set of excitation frequencies (such as those corresponding to wind, earthquake, etc.). The problem of maximizing the first eigenfrequency has been tackled by many authors by using the topological optimization method, [13–17]. However, in several structure designs, such as the deck of a bridge, we are not allowed to create holes. In that case, one can recourse to parametric optimization: improve the mechanical performances of the multilayered composite plate, such as the

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bending rigidity, the vibration resistance, the buckling resistance, the weight,..., with respect to its design variables (fiber orientations, kind of material, thickness of layers). When the design variables are of finite number, e.g., the fiber orientation in every ply is not constant but varies in the set $\{-45, 0, 45, 90\}$, the material is either orthotropic reinforced fibers or soft isotropic, the parametrization used is called *Discrete Material Optimization* (DMO) approach. Duvaut et al. [4], Hvejsel et al. [7], Setoodeh et al. [12] and Kennedy et al. [8] have been interested with the problem of maximizing the bending rigidity – or minimizing the compliance – of a multilayered plate, with respect to some of the above discrete design variables, using the DMO method. Gea et al. [6], Niu et al. [10] and Pedersen et al. [11] have studied some maximization problems, related to eigenfrequencies of multilayered plates, using the DMO method. Erdal et al. [5], Lund et al. [9] and Van Campen [29] have also used the DMO method for maximizing the buckling critical load of a laminated multi-material composite plate or shell. In all the above works, the optimal design structure with respect to one criterion is obtained, but has the drawback of being difficult to manufacture and thereby a very expensive design, [12,8]. Hence, if we require minimal cost of structures manufacturing, we should reduce the number of parameters such as the fiber orientation and the material are constant per layer and all layers have the same thickness. The latter design is obviously more easy to manufacture. Furthermore, looking for the optimal design with respect to only one criteria is not at all interesting. Indeed, an optimal design with respect to one criterion could be not optimal with respect to another criterion. The recourse to multiobjective optimization is, in my point of view, the solution to improve many performances in the same time. For instance, the multilayered plate should be very resistant to buckling phenomenon, besides having a good bending rigidity. To do this, we must maximize the smallest critical buckling load and minimize the compliance: the work of uniform transverse load p, applied to the upper surface of the plate, in the vertical displacement. Therefore, we are faced with a compromise.

In this paper, we suppose that the fiber orientation and the material are constant per layer and all layers have the same thickness. We deal with the three following problems. First, in order to check the performance of the simulated annealing algorithm, we minimize and maximize the compliance. Second, we look for maximizing, in the same time, the bending rigidity and the stability of the multilayered plate. For these purposes, Mindlin's plate model is considered, the six-node triangular finite element is used and the simulated annealing algorithm [18–21] as well as the Pareto front [22,23] method are implemented.

The layout of the paper is as follows. In Section 2, the problems of free vibration, of bending and of linear buckling are first of all recalled. Second, some interesting remarks, concerning the well posedness of the above optimization problems, are given. In section three, the used optimization algorithms are defined. Although the smallest eigenfrequency and the compliance are sufficiently regular with respect to θ, we did not use methods of the gradient type because our costs are not in general convex. We used metaheuristic methods: the simulated annealing algorithm [18,20,19,21] to approximate each global minimum of our costs, and the Pareto front method [22,23] for minimizing two criteria in the same time. Section 4 is devoted to two numerical experiments corresponding to a rectangular plate composed of eight symmetric layers. The plate is simply supported on the right side, clamped on the left side and free on the two other edges. Some conclusions and perspectives of this study are given in Section 5. The end of the paper is devoted to an Appendix which gives all the proofs of the regularity of the first cost.

2. Mathematical setting

Let

$$W_M = \{v = (v_3, r_1, r_2) \in H^1(\omega)^3;\ v_3 = 0 \text{ on } \partial\omega\},$$

be the space of kinematically admissible bending displacements, where v_3 denotes the deflection of the plate, r_1 and r_2 denote the rotations of its mid-plan, and $H_M = L^2(\omega)^3$. Let

$$V = \{v = (v_1, v_2) \in H^1(\omega)^2;\ v = 0 \text{ on } \gamma_0\},$$

be the space of kinematically admissible membrane displacements, where $v_\alpha,\ \alpha = 1, 2$, denote the displacements in the plane of the plate, and γ_0 is a portion of the boundary whose measure is not zero.

Let $(\underline{R}_{\alpha\beta\mu\nu})$ denote the planar rigidity tensor and $(S_{\alpha\beta\mu\nu})$ denote its inverse, i.e. the planar compliance tensor. The fourth-order symmetric tensors of bending rigidity and of membrane rigidity are respectively

$$D_{\alpha\beta\mu\nu}(\theta) = \int_{-\varepsilon}^{\varepsilon} x_3^2 \underline{R}_{\alpha\beta\mu\nu}(\theta)dx_3 \quad \text{and} \quad E_{\alpha\beta\mu\nu}(\theta) = \int_{-\varepsilon}^{\varepsilon} \underline{R}_{\alpha\beta\mu\nu}(\theta)dx_3.$$

They are assumed to satisfy the uniform ellipticity propriety: there exists positive constants α_1 and β_1 independent of θ, such that

$$\sum_{\alpha,\beta,\mu,\nu=1}^{2} D_{\alpha\beta\mu\nu}(\theta)\xi_{\mu\nu}\xi_{\alpha\beta} \geq \alpha_1 \sum_{\alpha,\beta=1}^{2} \xi_{\alpha\beta}^2 \quad \text{and} \quad \sum_{\alpha,\beta,\mu,\nu=1}^{2} E_{\alpha\beta\mu\nu}(\theta)\xi_{\mu\nu}\xi_{\alpha\beta} \geq \beta_1 \sum_{\alpha,\beta=1}^{2} \xi_{\alpha\beta}^2, \tag{2.1}$$

for all symmetric tensor of order two $(\xi_{\alpha\beta})$.