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Arbitrary decay rates of energy for a von Karman equation of memory type

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ABSTRACT

In this paper we consider a von Karman equation of memory type

$$u_{tt}+\Delta^2 u-\int_0^t g(t-s)\Delta^2 u(s)ds=[u,F(u)]$$

with clamped boundary condition. We establish a decay result of solutions without imposing the usual relation between a kernel function g and its derivative. This result generalizes earlier ones to an arbitrary rate of decay, which is not necessarily of an exponential or polynomial decay.

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1. Introduction

In this paper we consider the following von Karman system of memory type

$$u_{tt}+\Delta^2 u-\int_0^t g(t-s)\Delta^2 u(s)ds=[u,F(u)] \quad \text{in } \Omega\times\mathbb{R}^+, \tag{1.1}$$

$$\Delta^2 F(u)=-[u,u] \quad \text{in } \Omega\times\mathbb{R}^+, \tag{1.2}$$

$$u=\frac{\partial u}{\partial\nu}=0, \qquad F(u)=\frac{\partial F(u)}{\partial\nu}=0 \quad \text{on } \partial\Omega\times\mathbb{R}^+, \tag{1.3}$$

$$u(x,0)=u_0(x), \qquad u_t(x,0)=u_1(x) \quad \text{for } x\in\Omega, \tag{1.4}$$

where $\Omega\subset\mathbb{R}^2$ is a bounded domain with sufficiently smooth boundary $\partial\Omega$, $\nu=(\nu_1,\nu_2)$ is the outward unit normal vector to $\partial\Omega$, $x=(x_1,x_2)\in\Omega$, g is a kernel function which will be specified later and von Karman bracket is given by

$$[u,\phi]\equiv u_{x_1x_1}\phi_{x_2x_2}+u_{x_2x_2}\phi_{x_1x_1}-2u_{x_1x_2}\phi_{x_1x_2}.$$

From the physical point of view, problem (1.1)–(1.4) describes vertical oscillations of nonlinear plates subjects to large displacements. It is essential to note that this problem does not account for regularizing effects of rotational inertia $-\Delta u_{tt}$ which make the nonlinear term subcritical (see [1] for the discussion). A class of von Karman equations with dissipative effects has been studied by many authors [2–13]. As regards von Karman equation of the form

$$u_{tt}+\Delta^2 u+g(x,u_t)=[u,F(u)], \tag{1.5}$$

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it was discussed about the results on existence and asymptotic behavior (see e.g. [1,3–5,10] and references therein). When $g = 0$ in (1.5), Favini et al. [3] proved global existence, uniqueness and regularity of solutions for the equation with nonlinear boundary dissipation, moreover they showed the uniqueness of weak solutions by proving sharp regularity results of the Airy stress function, Park and Park [10] considered the existence of strong solutions and uniform decay rates for the equation with boundary memory condition. For the case $g \neq 0$ in (1.5), Horn and Lasiecka [4] investigated energy decay rates of weak solutions for the equation with $g(x, u_t) = b(x)u_t$ and nonlinear boundary dissipation.

With respect to a von Karman equation with rotational inertia and memory of the form

$$u_{tt} + \Delta^2 u - h\Delta u_{tt} - \int_0^t g(t-s)\Delta^2 u(s)ds = [u, F(u)], \tag{1.6}$$

several authors [8,11,13] investigated existence and stability of solutions. Munoz Rivera and Menzala [8] proved exponential and polynomial decay rates under the classical condition $g'(t) \le -\zeta g(t)$ and $g'(t) \le -cg^{1+\frac{1}{p}}(t)$, $p > 2$, respectively. For the problem related to these conditions, we refer [14]. Later, Park et al. [11] and Raposo and Santos [13] extended those results by proving general decay rates of the energy under the more general condition $g'(t) \le -\zeta(t)g(t)$, where $\zeta(t)$ is a nonincreasing and positive function (see [15–18,11] and references therein for the related problems). Though the presence of rotational inertia $-\Delta u_{tt}$ is quite legitimate from the physical point of view, it gives the amount of regularity necessary to compute via a suitable Liapunov functional. Recently, Cavalcanti et al. [19] considered problem (1.6) with $h = 0$ under the condition $g'(t) \le -H(g(t))$, where $H(s)$ is a given continuous, positive, increasing and convex function such that $H(0) = 0$. The feature of the work [19] is to provide wellposedness of both weak and regular solutions, and sharp and general decay rate estimates without accounting for regularizing effects of rotational inertia by pursuing the strategy introduced in [20–22].

On the other hand, Fabrizio and Polidoro [23] obtained exponential decay rates of solutions to a linear viscoelastic wave equation under the condition $g'(t) \le 0$ and $e^{\alpha t}g(t) \in L^1(0, \infty)$ for some $\alpha > 0$. Tatar [24] weakened this assumption as

$$g'(t) \le 0 \quad \text{and} \quad \zeta(t)g(t) \in L^1(0, \infty), \tag{1.7}$$

where $\zeta(t)$ is a nonnegative function, and established an arbitrary decay rate for a linear viscoelastic wave equation by introducing an appropriate new functional in the modified energy.

Inspired by these results, we improve earlier ones concerning exponential and polynomial decay rates for problem (1.1)–(1.4) by applying the condition (1.7). Since problem (1.1)–(1.4) is a nonlinear viscoelastic system, the estimates are more complicate. But we get the desired result by imposing some restriction on the initial data.

The remainder of the paper is organized as follows. In Section 2, we give some preliminaries related to problem (1.1)–(1.4). In Section 3, we prove an arbitrary decay result.

2. Preliminaries

In this section we review some notations about function spaces and preliminary results. We denote $(u, v) = \int_\Omega u(x)v(x)dx$. For a Hilbert space X, we denote $(\cdot, \cdot)_X$ and $\|\cdot\|_X$ the inner product and norm of X, respectively. For simplicity, we denote $\|\cdot\|_{L^2(\Omega)}$ by $\|\cdot\|$. Let λ_0 and λ be the smallest positive constants such that

$$\lambda_0\|u\|^2 \le \|\nabla u\|^2 \quad \text{and} \quad \lambda\|u\|^2 \le \|\Delta u\|^2 \quad \text{for } u \in H_0^2(\Omega). \tag{2.1}$$

Now we introduce relative results of the Airy stress function and von Karman bracket (see e.g. [2,1,3,4,7]).

Lemma 2.1. *If u, ϕ and ψ belong in $H^2(\Omega)$ and at least one of them belongs in $H_0^2(\Omega)$, then $([u, \phi], \psi) = ([u, \psi], \phi)$.*

Lemma 2.2. *If $u \in H^2(\Omega)$, then $\|F(u)\|_{W^{2,\infty}(\Omega)} \le c\|u\|^2_{H^2(\Omega)}$.*

Lemma 2.3. *If $u \in H^2(\Omega)$ and $\phi \in W^{2,\infty}(\Omega)$, then $\|[u, \phi]\| \le c\|u\|_{H^2(\Omega)}\|\phi\|_{W^{2,\infty}(\Omega)}$.*

As in [24], we impose the following conditions on the relaxation function g:

(G1) $g : \mathbb{R}^+ \to \mathbb{R}^+$ is a continuous, nonincreasing and almost everywhere differentiable function satisfying

$$g(0) > 0, \quad \int_0^\infty g(s)ds := l < 1. \tag{2.2}$$

(G2) There exists an increasing function $\zeta(t) > 0$ such that

$$\frac{\zeta'(t)}{\zeta(t)} := \eta(t) \quad \text{is a nonincreasing function and} \quad \int_0^\infty g(s)\zeta(s)ds < \infty. \tag{2.3}$$

We recall the existence results (see [19]):

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