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Weak form quadrature element method for accurate free vibration analysis of thin skew plates

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ABSTRACT

In this paper, a novel weak form quadrature element method (QEM) is proposed for accurate free transverse vibration analysis of thin isotropic skew plates with general boundary conditions. In the formulation of the stiffness matrix, the first and second order derivatives of the shape functions at integration points are computed explicitly by using the differential quadrature rule. This leads to a great simplicity in formulating an $N \times N$-node quadrature plate element and a large reduction of programming effort. Different from the existing weak form quadrature element method or differential quadrature finite element method, the element nodes can be either the same or different from the integration points. Convergence studies are performed. Free vibration of skew thin plates with various skew angles and different combinations of boundary conditions is analyzed. It is shown that although the assumed displacement field does not explicitly consider the bending moment singularities at the obtuse angles, the proposed QEM can yield accurate frequencies even for the thin isotropic skew plate with large skew angles.

1. Introduction

The skew plate is one of the common structural elements. Its behavior is important to structural engineers thus has been caused a lot of concerns. The topic of free vibration of thin and thick skew plates is a very common one and a vast body of literature exists [1–25]. Relative research work on thin skew plates has been well documented by Zhou and Zheng [12] and McGee [14], where over one hundred papers have been reviewed.

Due to the complicated mathematical structure of the partial differential equations, it is not an easy task to obtain closed form solutions for skew plates with general boundary conditions. Therefore, various continuous and numerical methods have been employed to obtain solutions. The methods include the Ritz method [2–5,14,15,17,24], the variational method (VM) [18], the mesh-less method with radial basis function (RBF) [25], the moving least squares-Ritz (MLS-Ritz) method [12], the strong form differential quadrature method (DQM), the harmonic differential quadrature method (HDQM) or the differential quadrature element method (DQEM) [6–11,20,23,26,27], the finite element method (FEM) [11,19] and the discrete singular convolution (DSC) algorithm [13,21,22].

For the thin isotropic skew plate when its skew angle is large, it is not an easy task to obtain accurate fundamental frequency, since strong bending moment singularities exist at the obtuse angles [2–5,14,15]. The above-mentioned

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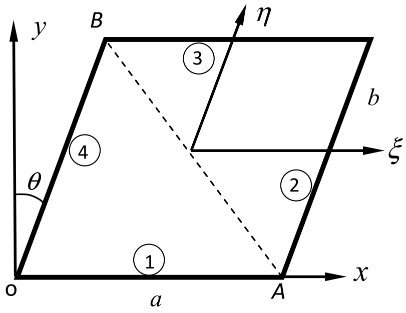

Fig. 1. Sketch of a skew plate.

approximate and numerical methods may encounter serious convergence problems if the assumed displacement does not explicitly consider the bending moment singularities at the obtuse angles [12,14]. It is shown that an accurate fundamental frequency cannot be obtained by various methods for the SSFF skew plate with a large skew angle ($\theta = 75^\circ$) [11]. Symbols SSFF stand for the plate that is simply supported at edges 1 and 2 and free at the other two edges, where the edge numbers are defined in Fig. 1. Compared to accurate upper bound solution given by Leissa et al. [5], the relative error for the fundamental frequency reaches up to 50% for the DQM and −20% for the MLS-Ritz, although these methods work very well for skew plates with all other combinations of boundary conditions [10–12]. Therefore, new or alternative numerical methods should be investigated to study the problem further.

Comparing to the strong form formulations, the weak form formulations would be better in handling irregular geometry, discontinuous distribution of load and the force boundary conditions. The zero bending moment conditions are natural boundary conditions, thus the weak form formulations based on the principle of minimum potential energy could weaken the effect of the bending moment singularity existing at the obtuse angles. Previous study [11] showed that the strong form DQM or the DQEM was very sensitive to the grid spacing when the strong bending moment singularity exists at the obtuse angles. Although the DSC algorithm can yield accurate lower order and high order mode frequencies [13,21,22], the method has some difficulties to deal with the force boundary conditions at free edges. Therefore, the weak form method, such as the weak form quadrature element method (QEM) or the *p*-version finite element method [28–31], is to be employed to solve the free vibration of thin skew plates.

The weak form quadrature element method is formulated based on the principle of minimum potential energy and the differential quadrature rule. The main features of the QEM are that non-uniformly distributed points are used as the element nodes and the strains at integration points are explicitly calculated by using the differential quadrature rule. Since the explicit formulas for the first and second order derivatives of the displacement with respect to space variables are not needed, the formulations of stiffness matrix of various weak form quadrature elements with different number of nodes are extremely simple [31] and a general formulation of the element with variable nodes can be provided. Besides, various formulas of computing the weighting coefficients of the first and second order derivatives in the DQM can be used in the quadrature element formulations. This is based on the fact that different displacement functions can be used to calculate the derivatives according to the criteria for selection of displacement functions in the conventional FEM. Since the formulation is general, therefore, it is easy to achieve a *p*-version adaptive by changing the number of the element nodes according to the accuracy requirement [26,32].

In the early version of QEM [28], the element can be with any kind of nodes, such as uniformly distributed nodes, Chebyshev–Lobatto–Gauss (CLG) nodes and Gauss–Lobatto–Legendre (GLL) nodes. However, the derivations are much complicated, since it requires computing the inverse of a full Vandermonde matrix. Thus the number of element nodes is usually small and fixed. To remove this deficiency and directly use the differential quadrature rule to compute the strains at integration points, the elemental nodes should be the same as the integration points. Thus GLL points are uniquely used as the element nodes and the GLL quadrature is used in obtaining the stiffness matrix [29,30]. In such cases, there is no much difference between the QEM and the time domain spectral element method (SEM), especially when the element does not have derivative degrees of freedom. In principle, the QEM and the SEM are the same as the high order finite element method. The efficiency and accuracy of the method has been demonstrated when one or more than one element is used to represent the solution domain [29,30].

More recently, present authors proposed a more general quadrature element method [31]. Expanded Chebyshev (EC) points are used as the element nodes and Gauss quadrature is used to obtain the stiffness matrix. Since the element nodal points can be different from the integration points thus the method is general and flexible for developing elements based on different theories. Some other advantages of the new method are that it can be used to perform the reduced integration often adopted in commercial software or to calculate strains at locations other than the nodal points if it is required. The proposed method in [31] is, however, only approximate if the shape functions are Hermite interpolation functions.

The objective of the present investigation is to propose a novel weak form quadrature element method (QEM) for accurate free transverse vibration analysis of thin anisotropic rectangular plates and isotropic skew plates. Three sets of element