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Optimal control of feedback control systems governed by hemivariational inequalities[☆]

Yong Huang [a], Zhenhai Liu [b,c,*], Biao Zeng [d]

[a] Department of Mathematics, Baise University, Baise 533000, Guangxi Province, PR China

[b] Guangxi Key Laboratory of Universities Optimization Control and Engineering Calculation, Guangxi University for Nationalities, Nanning 530006, Guangxi Province, PR China

[c] College of Sciences, Guangxi University for Nationalities, Nanning 530006, Guangxi Province, PR China

[d] Faculty of Mathematics and Computer Science, Jagiellonian University, Institute of Computer Science, ul. Łojasiewicza 6, 30-348 Krakow, Poland

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ABSTRACT

This paper is mainly concerned with the feedback control systems governed by evolution hemivariational inequalities. By using the properties of multimaps and Clarke's subdifferential, we formulate some sufficient conditions to guarantee the existence result of feasible pairs of the feedback control systems. We also present an existence result of optimal control pairs for an optimal control problem. We emphasize that our results cannot be obtained straightforwardly from the previous works.

1. Introduction

Let H be a separable Hilbert space, $\langle\cdot,\cdot\rangle_H$ the inner product of H. $A : D(A) \subseteq H \to H$ is the infinitesimal generator of a uniformly bounded compact C_0-semigroup $\{T(t)\}_{t>0}$ on H. Let V be a reflexive Banach space, $u : [0,T] \to V$ a control function and $B : V \to H$ a bounded linear operator. The notation $F^0(t,\cdot;\cdot)$ stands for the generalized Clarke's directional derivative (cf. [1]) of a locally Lipschitz function $F(t,\cdot) : H \to \mathbb{R}$. In this paper, we firstly study the existence of solutions of the following evolution hemivariational inequalities:

$$\begin{cases} \langle -x'(t) + Ax(t) + Bu(t), v\rangle_H + F^0(t, x(t); v) \geq 0, & \text{a.e. } t \in [0,T],\ \forall v \in H, \\ x(0) = x_0 \in H. \end{cases} \tag{1.1}$$

Next, we shall be concerned with the existence of feasible pairs of the following feedback control systems:

$$\begin{cases} \langle -x'(t) + Ax(t) + Bu(t), v\rangle_H + F^0(t, x(t); v) \geq 0, & \text{a.e. } t \in [0,T],\ \forall v \in H, \\ u(t) \in U(t, x(t)), \\ x(0) = x_0 \in H, \end{cases} \tag{1.2}$$

where $U : [0,T] \times H \to P(V)$ is a multimap.

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* Corresponding author. Tel.: +86 771 3265663, +86 771 3260370.
E-mail addresses: yonghuang99@126.com (Y. Huang), zhhliu@hotmail.com (Z. Liu), zengbiao316711602@163.com (B. Zeng).

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Hemivariational inequalities have important applications in mechanics and engineering, especially in nonsmooth analysis and optimization (see [2–16]). With the development of the study of hemivariational inequalities, some scholars have begun to pay their attentions to the optimal control problems for hemivariational inequalities. In particular, Haslinger and Panagiotopoulos [17] obtained the existence of optimal control pairs for a class of coercive hemivariational inequalities. In [18], Migorski and Ochal investigated the optimal control problems for the parabolic hemivariational inequalities. J.Y. Park and S.H. Park [15,16] showed the existence of optimal control pairs to the hyperbolic linear systems. In [19,20], Tolstonogov paid his attention to the optimal control problems for differential inclusions with subdifferential type.

Feedback control systems are ubiquitous around us, including trajectory planning of a robot manipulator, guidance of a tactical missile toward a moving target, regulation of room temperature, and control of string vibrations. Optimal feedback control of semilinear evolution equations in Banach spaces has been studied [21–24]. However, the study for the optimal control of feedback control systems described by evolution hemivariational inequalities is still untreated topic in the literature and this fact is the motivation of the present work. The aim of this paper is study the existence result of feasible pairs of feedback optimal control systems for evolution hemivariational inequalities. By using the properties of multimaps and Clarke's subdifferential, a new set of sufficient conditions are formulated to guarantee our main results.

The rest of this paper is organized as follows. In the next section, we will introduce some useful preliminaries and physical models. In Section 3, some sufficient conditions and techniques are established for the existence of feasible pairs of problem (1.2). We first study the existence of solutions of (1.1) by a fixed point theorem of multimaps. In Section 4, we will study the optimal control of problem (1.2).

2. Preliminaries and physical models

In this section, we first introduce some basic preliminaries which are used throughout this paper. The norm of the Hilbert space H will be denoted by $\|\cdot\|_H$. Let $J=[0,T]$. For a uniformly bounded C_0-semigroup $\{T(t)\}_{t\geq 0}$, there exists $M>0$ such that $\sup_{t\in[0,\infty)}\|T(t)\|\leq M$ [25]. Let $C(J,H)$ denote the Banach space of all continuous functions from J into H with the norm $\|x\|_C=\sup_{t\in J}\|x(t)\|_H$, $L^2(J,H)$ denote the Banach space of all Bochner L^2-integrable functions from J into H with the norm $\|x\|_{L^2}=\left(\int_0^T\|x(s)\|^2ds\right)^{\frac{1}{2}}$.

Let us recall some definitions and properties about multimaps. For more details we refer to [26–31].

Let X and Y be two topological spaces. Denote by $P(Y)$ $[C(Y), K(Y), K_v(Y)]$ the collections of all nonempty [respectively, nonempty closed, nonempty compact, nonempty compact convex] subsets of Y. A multimap $F:J\to C(X)$ is said to be measurable, if $F^{-1}(Q):=\{x\in J|F(x)\cap Q\neq\emptyset\}\in\mathcal{L}$ for every closed set $Q\subset X$, where $\mathcal{L}$ denotes the σ-field of Lebesgue measurable sets on J. Every measurable multimap F admits a measurable selection $f:J\to X$, i.e., f is measurable and $f(t)\in F(t)$ for a.e. $t\in J$. A multimap $F:X\to C(Y)$ is said to be upper semicontinuous (or u.s.c. for short), if for every open subset $D\subset Y$ the set $F_+^{-1}(D)=\{x\in X: F(x)\subset D\}$ is open in X; weakly u.s.c., if $F:X\to C(Y_w)$ is u.s.c., where Y_w is the space Y equipped with a weak topology. A multimap $F:X\to C(Y)$ is said to be closed if its graph $Gr(F):=\{(x,y)\in X\times Y: x\in X, y\in F(x)\}$ is a closed subset of $X\times Y$; compact, if F maps bounded sets of X into relatively compact sets in Y.

We have the following important property for multimaps.

Lemma 2.1 ([29, Theorem 1.1.12]). *Let X and Y be metric spaces and $F:X\to K(Y)$ a closed compact multimap. Then F is u.s.c.*

Definition 2.2 ([23]). Let X be a Banach space and Y be a metric space. Let $F:X\to P(Y)$ be a multimap. We say F possesses the Cesari property at $x_0\in X$, if

$$\bigcap_{\delta>0}\overline{co}F(O_\delta(x_0))=F(x_0),$$

where $\overline{co}D$ is the closed convex hull of D, $O_\delta(x)$ is the δ-neighborhood of x. If F has the Cesari property at every point $x\in Z\subset X$, we simply say that F has the Cesari property on Z.

Lemma 2.3 ([23, Proposition 4.2]). *Let X be a Banach space and Y be a metric space. Let $F:X\to P(Y)$ be u.s.c. with convex and closed valued. Then F has the Cesari property on X.*

Now, let us proceed to the definition of the Clarke's subdifferential for a locally Lipschitz function $j:X\to\mathbb{R}$, where X is a Banach space and X^* is the dual space of X (one can see [1,32,14]). We denote by $j^0(x;v)$ the Clarke's generalized directional derivative of j at the point $x\in X$ in the direction $v\in X$, that is

$$j^0(x;v):=\limsup_{\lambda\to 0^+,\ \zeta\to x}\frac{j(\zeta+\lambda v)-j(\zeta)}{\lambda}.$$

Recall also that the Clarke's subdifferential or generalized gradient of j at $x\in X$, denoted by $\partial j(x)$, is a subset of X^* given by

$$\partial j(x):=\{x^*\in X^*: j^0(x;v)\geq\langle x^*,v\rangle,\ \forall v\in X\}.$$