Computers and Mathematics with Applications 69 (2015) 1329–1341

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

An analytical solution for free and forced vibration of a piezoelectric laminated plate coupled with an acoustic enclosure

Mehran Shahraeeni [a,*], Rezgar Shakeri [b], Seyyed Mohammad Hasheminejad [a]

[a] School of Mechanical Engineering, Iran University of Science and Technology, Narmak, Tehran 16846-13114, Iran
[b] School of Railway Engineering, Iran University of Science and Technology, Narmak, Tehran 16846-13114, Iran

ARTICLE INFO

Article history:
Received 8 July 2014
Received in revised form 8 January 2015
Accepted 12 March 2015
Available online 7 April 2015

Keywords:
Acoustic cavity
Helmholtz equation
Modal approach
Piezoelectric laminated plate
Time domain analysis

ABSTRACT

This paper deals with analytical vibro-acoustic modeling of a three-layered piezoelectric laminate coupled with an acoustic enclosure. Equation of the motion of the simply supported laminate is derived using the classical lamination theory (CLT) and pressure distribution inside the enclosure is obtained from the linear acoustic wave equation. Piezoelectric layers are assumed to be fully electroded and electric potential distribution in the piezoelectric layers satisfies both Maxwell's equation and closed circuit condition on major surfaces of the piezoelectric layers. In order to study the free and forced vibratory characteristics of the coupled system, the time dependent motion equation of the coupled system transformed into frequency domain and Laplace domain and reduced to a set of ordinary differential equations, based on the eigenfunction expansion method. Afterwards, the effect of the cavity depth upon natural frequencies and dynamic response of the presented model is examined. To ensure the accuracy of the presented model, results are compared with those obtained from commercial finite element analysis software.

1. Introduction

Identifying the coupled system natural frequencies and dynamic behavior of systems in the presence of acoustic-structure interaction is one of the most important issues in the engineering design of buildings, road vehicles and aircraft. Vibrating components of the structures play a major role in the creation of high sound pressure levels (SPL) within the interior enclosed spaces which, in turn, result in a noticeable decline in the human comfort. In recent years, there has been an increased interest in active noise and vibration cancellation techniques that rely on adding smart materials to the host structure. Smart materials such as piezoelectric ceramics and polymers contribute to noteworthy changes in natural frequencies and dynamic behavior of the host systems; hence, they are commonly used for control purposes.

Over the last 50 years, several analytical and semi-analytical formulations have been presented for predicting the vibro-acoustic behavior of cavity–plate systems as well as free and forced vibration characteristics of laminated plates embedded with piezoelectric layers. Here, only the most significant studies with respect to the current research are briefly reviewed. In 1963, Lyon [1] was the first to study noise reduction in a rectangular enclosure with one flexible wall. The cavity was exposed to external pressure waves, and an approximate energy method was implemented to estimate the noise reduction

* Corresponding author. Tel.: +98 915 1217717.
E-mail addresses: shahraeeni@mecheng.iust.ac.ir, me.shahraeeni@yahoo.com (M. Shahraeeni), rezgarshakeri@gmail.com (R. Shakeri), hashemi@iust.ac.ir (S.M. Hasheminejad).

http://dx.doi.org/10.1016/j.camwa.2015.03.022
0898-1221/

in various frequency ranges. Dowell et al. [2] proposed an analytical solution for the velocity potential inside an enclosure with one flexible wall in terms of the Fourier series. In 1965, Pretlove [3] proposed an analytical solution for the free vibration of a simply supported panel backed by a rectangular cavity based on the work done by Dowell et al. He used only the symmetric–symmetric in vacuo normal modes of the panel and investigated the effect of the cavity depth on the coupling intensity. The following year, Pretlove [4] studied the forced vibration of the former system in frequency domain with the temporal Fourier transform. Fahy [5] considered the cavity and plate as two separate subsystems interacting to exchange energy and utilized their uncoupled eigenfunctions to predict the vibrational and acoustic properties of the coupled system. The intrinsic concept of this method, which is called the modal approach to statistical energy analysis (SEA), was introduced independently by Lyon and Smith [6,7] in the late 1950s. Narayanan [8] analytically studied sound transmission through sandwich panels with viscoelastic cores into enclosures. A comprehensive literature review has been given by Pan et al. [9] on the panel–cavity studies performed since 1990. Atalla and Tournour [10] studied the effect of the modal truncation on the convergence of the modal coupling method and improved its convergence by pseudostatic corrections for both the cavity and the structure. Luo et al. [11] studied the response of irregular cavities under harmonic excitation, based on Green's function theorem. Gorman et al. [12] proposed an analytical solution based on the implementation of the Galerkin technique and an approximate solution based on modal energy analysis to study the vibratory behavior of a thin circular plate coupled with a cylindrical enclosure. Rocha et al. [13] modeled an aircraft cabin as a cylindrical acoustic cavity, filled with air, and developed an analytical framework to predict the structural vibration of the fuselage and sound pressure levels in the fuselage under turbulent boundary layer excitation. Du et al. [14] investigated the vibro-acoustic behavior of an elastically restrained panel coupled with an enclosure and proposed an improved Fourier series to enforce the velocity continuity condition near or on the fluid–structure interface. In order to investigate the impact of adding distributed masses and internal sound sources on the vibratory behavior of panel–cavity systems, Boltežar et al. [15] used the modal coupling theory in conjunction with the Rayleigh–Ritz method and conducted several experiments to validate their results. Jin et al. [16] employed Chebyshev series to analyze the vibro-acoustic features of enclosures with general wall impedance coupled with elastically restrained plates. A recent study by Younesian and Sadri [17] involved studying the nonlinear free vibration of plate–cavity systems with the Von-Karman plate theory and variational iteration method. Lee [18] developed a theory for laminated piezoelectric plates based on Kirchhoff plate theory in 1990. Tzou et al. [19] examined models of plates with segmented piezoelectric sensor and actuator layers and discussed the enhancement of observability and controllability of the system due to segmentation. Vel and Batra [20] developed an analytical formulation to obtain the three dimensional deformations of a simply supported laminated rectangular plate embedded with piezoelectric actuators. Wang et al. [21,22] proposed a quadratic potential distribution function for piezoelectric layers to satisfy Maxwell's equation and studied the free vibration of a two-dimensional thin circular plate surface bonded by piezoelectric layers with closed and open circuit conditions and showed that for the open circuit condition, compared with closed circuit condition, the natural frequencies are noticeably affected by electrical features of piezoelectric material. Pietrzakowski [23] analytically investigated the natural frequencies of thin and moderately thick composite plates comprised of piezoelectric sensor and actuator layers. Chen et al. [24] and Larbi and Deü [25] proposed analytical solutions for free vibration analysis of laminated piezoelectric hollow cylinders filled with compressible fluid. Balachandran et al. [26,27] carried out analytical and experimental studies for controlling noise in a three-dimensional cavity with one flexible wall, which was clamped along all the edges, using piezoelectric patches bonded symmetrically to the top and bottom of panel. Jin et al. [28] presented an analytical model for active noise control of a cabin-like enclosure comprised of two simply supported plates using Lead zirconate titanate (PZT) patches for actuation and sensing.

From the authors' knowledge, free and forced vibration of piezoelectric laminated plates coupled with rectangular acoustic cavities has not been studied analytically and there is a lack of analytical models to provide fundamental insights for characterizing the acoustical and electromechanical properties of rectangular acoustic cavities coupled with piezoelectric laminates. To address the issue, we develop a novel analytical model in this paper to study the vibratory behavior of piezoelectric laminates when they are in contact with a finite acoustic medium. Significant contributions of this work include development of a detailed and straightforward analytical framework for free and forced vibration analysis of a piezoelectric laminate coupled with a rectangular acoustic cavity and providing a design reference model for active control of sound and vibration in cavity–laminate systems in time domain.

2. Theoretical formulation

2.1. Equation of the motion of a three-layered piezoelectric laminate

Displacement field in the plate (u, v, w) can be expressed by the Kirchhoff hypotheses as

$$u = -z\frac{\partial w}{\partial x}, \qquad v = -z\frac{\partial w}{\partial y}, \quad w = w\,(x, y)\,, \tag{1}$$

where u and v are the displacements in x and y axes, respectively, whereas w is the transverse displacement along the z axis. The strain field in terms of the displacements can be written as follows.

$$\boldsymbol{\varepsilon} = \begin{bmatrix}\varepsilon_x & \varepsilon_y & \gamma_{xy}\end{bmatrix}^T = \begin{bmatrix}\dfrac{\partial u}{\partial x} & \dfrac{\partial v}{\partial y} & \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x}\end{bmatrix}^T = -z\begin{bmatrix}\dfrac{\partial^2 w}{\partial x^2} & \dfrac{\partial^2 w}{\partial y^2} & 2\dfrac{\partial^2 w}{\partial x \partial y}\end{bmatrix}^T. \tag{2}$$