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An efficient and accurate implementation of the Localized Regular Dual Reciprocity Method

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ABSTRACT

In this work we present an efficient and accurate implementation of the LRDRM. This integral domain decomposition method exploits two advantages: first, it imposes the boundary conditions at the Local RBF interpolation. Second, the integrals to compute are always regular. The approximation of the derivative of the field variable is computed in a posteriori way, directly differentiating the Local RBF interpolation or the local integral equation. The efficient and accurate behaviour of this method are demonstrated by performing numerical examples, with special emphasis on a 1D benchmark convective-diffusion equation. Results for 2D convection–diffusion, 2D Helmholtz and 2D Poisson equations are also presented.

1. Introduction

The boundary element method (BEM) is now a well-established numerical technique in engineering. The basis of this method is to transform the original partial differential equation (PDE), or system of PDEs that define a given physical problem, into an equivalent integral equation (or system) by means of the corresponding Green's second identity and its fundamental solution, i.e. Green's integral representation formula. In this way some or all of the field variables and their derivatives are only necessary to be defined at the boundary.

Further increase in the number of applications of the BEM has been hampered by the need to operate with relatively complex fundamental solutions or by the difficulties encountered when these solutions cannot be expressed in a closed form. In the BEM formulation of this kind of problems, it is common to use an integral representation formula based upon a PDE with known closed-form fundamental solution, and express the remaining terms of the original equation as domain integrals. It is known that in these cases the BEM is in disadvantage in comparison with the classical domain schemes, such as the Control Volume (CV) and the Finite Element method (FEM). In the early BEM analysis the evaluation of domain integrals was done using cell integration, a technique which, while effective and general, made the approach too costly computationally due to the successive integration at each cell required for each of the surface collocation points. Although good results can be obtained using the cell integration technique, this approach for certain applications is several orders of magnitude more time consuming than classical domain methods. This computational cost mainly depends on the fact that the solution at each surface or internal point must involve the evaluation of the corresponding surface integrals over the problem boundaries.

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Several methods have been developed in the literature to take domain integrals to the boundary in order to eliminate the need for internal cells (boundary-only BEM formulations). One of the most popular methods to date is the dual reciprocity method (DRM) introduced by Nardini and Brebbia [1]. In the DRM, the unknown densities of the corresponding domain integrals are interpolated by a Radial Basis Function (RBF) scheme, and by applying Green's second identity to a convolution integral of a particular solution and the fundamental solution, the domain integrals are converted into equivalent surface integrals. However, the DRM approach has the same computational limitations than the cell integration scheme, since very large fully populated matrix systems are obtained. It is important to mention that the DRM approximation is an alternative approach to evaluate domain integrals by defining global domain interpolations and only evaluating surface integrals, but still a domain integration scheme.

When dealing with the BEM for large problems, with or without closed form fundamental solution, it is frequently used a domain decomposition technique, in which the original domain is divided into subdomains, and on each of them the full integral representation formulae are applied. At the interfaces of the adjacent subdomains the corresponding full-matching conditions are imposed (local matrix assembly), as is required in the CV and FEM methods, for which it is necessary to define subdomains or elements connectivity. However, in contrast with the CV and FEM methods, which integral representations of the original PDE are based on weighted residual approximation, in the BEM technique Green's integral representation formula is an exact representation of the original PDE at each integration subdomain.

While the BEM matrices, which arise in the single domain formulation, are fully populated, the subdomain formulation leads to block banded matrix systems with one block for each subregion and overlaps between blocks when subdomains have a common interface. In the limit of a very large number of subdomains, the resulting internal mesh pattern looks like a finite element grid. The implementation of the subdomain BEM formulation in this limiting case, i.e. a very large number of subdomains, including cells integration at each subdomain has been called by Taigbenu and collaborators as the Green element method (GEM) (see [2]). A similar approach based on large number of subdomains but using the DRM to evaluate the domain integrals at each subdomain, instead of cell integration, has been referred by Popov and Power [3] as the Dual Reciprocity Multi Domain approach (DRM-MD), for more details see Portapila and Power [4]. As previously commented, the most attractive aspect of this type of local BEM approach at the subdomain level is the use of an exact integral representation formula of the original PDE instead of a weighted residual approximation. However, the numerical efficiency of this type of local BEM approaches is still behind of those classical domain numerical schemes. For this reason in recent years significant efforts have been given to the improvement of this type of local BEM approaches.

As has been the case in the FEM, see Atluri and Zhu [5], meshless formulations of local BEM approaches, see Zhu et al., [6], are attractive and efficient techniques to improve the performance of local BEM schemes. As in the meshless FEM, in the meshless BEM the integral representation formulae are applied at local internal integration subdomains (or Green's elements) embedded into interpolation stencils that are heavily overlapped. In this type of approach the continuity of the field variables is satisfied by the interpolation functions avoiding the local connectivity between subdomains or elements needed to enforce the matching conditions between them. Different interpolation schemes can be employed at the interpolation stencils, being the moving least squares shape functions and RBF interpolations the most popular approaches used in the literature. A major advantage of the meshless local BEM formulations in comparison with the classical BEM multi domain decomposition approaches, as the GEM and the DRM-MD, is that the resulting integrands of the integral representation formulae are all regular, instead of singular, since the collocation points are always selected inside the integration subdomain. The Localized Regular Dual Reciprocity Method considered in this work is one of those meshless local Boundary Integral Equation approaches, where RBFs are used as interpolation functions at the local stencils.

In recent years, the theory of radial basis functions (RBFs) has undergone intensive research and enjoyed considerable success as a technique for interpolating multivariable data and functions. The idea of introducing RBF interpolation to improve the accuracy of a classical numerical scheme has been employed by Wright and Fornberg [7]. In this work they generalized compact Finite Difference (FD) formulas for scattered nodes and RBFs, achieving the goal of keeping the number of stencil nodes small without a similar reduction in accuracy. They analyse the accuracy of these new compact RBF-FD formulas by applying them to model problems involving the Laplace linear differential operator and they study the effects of the shape parameter that arises in the infinitely smooth RBFs, multiquadric and Gaussian functions.

In [8] a modified Control Volume (CV) method which uses a RBF interpolation to improve the prediction of the flux accuracy at the faces of the CV is presented. This method is also more flexible than the classical CV formulations because the boundary conditions are explicitly imposed in the interpolation formula, without the need for artificial schemes (e.g. utilizing dummy cells).

In the Local Boundary Integral Element Methods (LBEM or LBIEM) the solution domain is covered by a series of small and heavily overlapping local interpolation stencils, where a direct interpolation of the field variables is used to approximate the densities of the integral operator, and the boundary conditions of the problem are imposed at the integral representation formula; i.e. at the global system of equations, resulting in the evaluation of the corresponding weakly and singular surface integrals and if it is the case regular domain integrals, over each of the integration subdomains including those in contact with the problem boundary [9,6,10–13]. In this type of approach, the domains of integration usually are defined over several stencils, resulting in highly overlapping integration subdomains, in addition to the overlapping of interpolation stencils. Both polynomial moving least squares (MLS) approximation and direct RBF interpolations have been previously used in the LBEM as local interpolation algorithms.