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Certain results for the 2-variable Apostol type and related polynomials

Subuhi Khan [a,*], Ghazala Yasmin [b], Mumtaz Riyasat [a]

[a] Department of Mathematics, Aligarh Muslim University, Aligarh, India

[b] Department of Applied Mathematics, Faculty of Engineering, Aligarh Muslim University, Aligarh, India

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ABSTRACT

In this article, the 2-variable general polynomials are taken as base with Apostol type polynomials to introduce a family of 2-variable Apostol type polynomials. These polynomials are framed within the context of monomiality principle and their properties are established. Certain summation formulae for these polynomials are also derived. Examples of some members belonging to this family are considered and numbers related to some mixed special polynomials are also explored.

1. Introduction and preliminaries

Generalized and multivariable forms of the special functions of mathematical physics have witnessed a significant evolution during the recent years. In particular, the special polynomials of two variables provided new means of analysis for the solution of large classes of partial differential equations often encountered in physical problems. Most of the special functions of mathematical physics and their generalizations have been suggested by physical problems.

To give an example, we recall that the 2-variable Hermite Kampé de Feriet polynomials (2VHKdFP) $H_n(x,y)$ [1] defined by the generating function

$$\exp(xt+yt^2)=\sum_{n=0}^{\infty}H_n(x,y)\frac{t^n}{n!} \tag{1.1}$$

are solutions of the heat equation

$$\frac{\partial}{\partial y}H_n(x,y)=\frac{\partial^2}{\partial x^2}H_n(x,y), \tag{1.2a}$$

$$H_n(x,0)=x^n. \tag{1.2b}$$

The higher order Hermite polynomials, sometimes called the Kampé de Feriet polynomials of order m or the Gould–Hopper polynomials (GHP) $H_n^{(m)}(x,y)$ defined by the generating function [2, p. 58 (6.3)]

$$\exp(xt+yt^m)=\sum_{n=0}^{\infty}H_n^{(m)}(x,y)\frac{t^n}{n!} \tag{1.3}$$

* Corresponding author.
E-mail addresses: subuhi2006@gmail.com (S. Khan), ghazala30@gmail.com (G. Yasmin), mumtazrst@gmail.com (M. Riyasat).

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are solutions of the generalized heat equation [3]

$$\frac{\partial}{\partial y} f(x,y) = \frac{\partial^m}{\partial x^m} f(x,y), \tag{1.4a}$$

$$f(x,0) = x^n. \tag{1.4b}$$

Also, we note that

$$H_n^{(2)}(x,y) = H_n(x,y), \tag{1.5}$$

$$H_n(2x,-1) = H_n(x), \tag{1.6}$$

where $H_n(x)$ are the classical Hermite polynomials [4].

Next, we recall that the 2-variable Laguerre polynomials (2VLP) $L_n(y,x)$ [5, p. 150] are defined by the generating function

$$e^{xt} C_0(yt) = \sum_{n=0}^{\infty} L_n(y,x) \frac{t^n}{n!}, \tag{1.7}$$

where $C_0(y)$ denotes the 0th order Tricomi function [4]. These polynomials are the natural solutions of the equation

$$\frac{\partial}{\partial x} L_n(y,x) = -\left(\frac{\partial}{\partial y} y \frac{\partial}{\partial y}\right) L_n(y,x), \tag{1.8a}$$

$$L_n(y,0) = \frac{(-y)^n}{n!}, \tag{1.8b}$$

which is a kind of heat diffusion equation of Fokker–Planck type and is used to study the beam life-time due to quantum fluctuation in storage rings [6].

The 2-variable generalized Laguerre polynomials (2VGLP) ${}_mL_n(y,x)$ are defined by the following generating function [7, p. 214 (30)]:

$$e^{xt} C_0(-yt^m) = \sum_{n=0}^{\infty} {}_mL_n(y,x) \frac{t^n}{n!}. \tag{1.9}$$

In particular, we note that

$${}_1L_n(-y,x) = L_n(y,x), \tag{1.10}$$

$$L_n(y,1) = L_n(y), \tag{1.11}$$

where $L_n(y)$ are the classical Laguerre polynomials [4].

Further, we recall that the truncated exponential polynomials (TEP) $e_n(x)$ [4] defined by the series

$$e_n(x) = \sum_{k=0}^{n} \frac{x^k}{k!}, \tag{1.12}$$

are the first $(n+1)$ terms of the Maclaurin series for e^x. The generating function for $e_n(x)$ is given as [8, p. 596 (4)]

$$\frac{e^{xt}}{(1-t)} = \sum_{n=0}^{\infty} e_n(x) \frac{t^n}{n!}. \tag{1.13}$$

These polynomials appear in many problems of optics and quantum mechanics and also play an important role in the evaluation of integrals involving product of special functions.

The possibility of extending the definition of $e_n(x)$ is suggested within the context of its applications. We consider the 2-variable truncated exponential polynomials (2VTEP) (of order r) $e_n^{(r)}(x,y)$, which are defined by the generating function [9, p. 174 (30)]

$$\frac{e^{xt}}{(1-yt^r)} = \sum_{n=0}^{\infty} e_n^{(r)}(x,y) t^n. \tag{1.14}$$

In particular, we note that

$$e_n^{(2)}(x,y) = n!\, {}_{[2]}e_n(x,y), \tag{1.15}$$

$${}_{[2]}e_n(x,1) = {}_{[2]}e_n(x), \tag{1.16}$$

where ${}_{[2]}e_n(x,y)$ denotes the 2-variable truncated exponential polynomials (2VTEP) [8].