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Existence and multiplicity of periodic solutions for a class of second-order Hamiltonian systems☆

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ABSTRACT

We study the existence and multiplicity of periodic solutions of the following second-order Hamiltonian system

$$\ddot{u}(t) + \nabla[-K(t, u(t)) + W(t, u(t))] = 0.$$

The existence of a nontrivial periodic solution is obtained when ∇W is asymptotically linear at infinity, and the existence of infinitely many periodic solutions is also obtained when ∇W is superlinear.

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1. Introduction and main results

Consider the second-order Hamiltonian systems

$$\ddot{u}(t) + \nabla F(t, u(t)) = 0, \tag{1.1}$$

where $t \in \mathbb{R}$, $u \in \mathbb{R}^N$ and $F : \mathbb{R} \times \mathbb{R}^N \to \mathbb{R}$ satisfies:

(F1) $F(t, x) = -K(t, x) + W(t, x)$, $K, W \in C^1(\mathbb{R} \times \mathbb{R}^N, \mathbb{R})$ and are T-periodic in its first variable with $T > 0$.

Recently the existence and multiplicity of periodic solutions for system (1.1) have been studied in many papers via critical point theory; see [1–20]. In a pioneering work, Rabinowitz [13] established the existence of periodic solutions for system (1.1) under the well known Ambrosetti–Rabinowitz condition: there exist some $\mu > 2$ such that

$$0 < \mu F(t, x) \le (\nabla F(t, x), x) \tag{AR}$$

for all $t \in [0, T]$ and $x \in \mathbb{R}^N \setminus \{0\}$. Since then, the (AR)-condition has been used extensively to study superlinear Hamiltonian systems, for example, see [21–23,11,20] and references therein. Under the usual (AR)-condition, it is easy to show that the energy functional associated with the system has the Mountain Pass geometry and satisfies the (PS)-condition. Recently, Fei [6] and Tao and Tang [16] studied the existence of periodic solutions for Hamiltonian systems under a different superlinear conditions.

However, since (AR) does not hold for asymptotically linear cases, the problem becomes more delicate and hence some topology tools are involved. Using a Morse theory for strongly indefinite functionals, Abbondandolo [1] obtained periodic solutions for a class of first-order systems which are T-resonant at infinity. Fei [7] obtained, via a Maslov-type index, a non-trivial periodic solution for first-order systems which are resonant at infinity; the results were generalized later by Su [14].

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See also Fei [8] for multiple results. First-order systems which are resonant both at zero and at infinity were considered in Szulkin and Zou [15], Guo [9], Fei [5]. Degiovanni and Olian Fannio [4] proved multiple periodic solutions for autonomous asymptotically linear first-order Hamiltonian systems, on the basis of spectral properties of the matrices $H_{zz}(\infty)$ and $H_{zz}(0)$, where H is the Hamiltonian of the system and H_{zz} denotes its Hessian.

In recent paper [19], Zhao, Chen and Yang studied the existence of periodic solutions of system (1.1) with asymptotically linear function $\nabla F(t,x)$. Unlike in the works mentioned above, the behavior of $\nabla F(t,x)$ at infinity is like that of a function $V_\infty(t)x$, where $V_\infty(t)$ is a real valued function but not a matrix valued function. In detail, they obtained the following theorem.

Theorem A (*[19]*). *Assume that F satisfies* (F1), *and that K and W satisfy the following conditions:*

(K1) *There exist two constants $b_1 > 0$ and $b_2 > 0$ such that for all $(t,x) \in [0,T] \times \mathbb{R}^N$*

$$b_1|x|^2 \leq K(t,x) \leq b_2|x|^2;$$

(K2) $K(t,x) \leq (x, \nabla K(t,x)) \leq 2K(t,x)$ *for all* $(t,x) \in [0,T] \times \mathbb{R}^N$;

(W1) $\nabla W(t,x) = o(|x|)$, *as* $|x| \to 0$ *uniformly for* $t \in [0,T]$;

(W2) *There exists a function $V_\infty \in L^\infty([0,T],\mathbb{R})$ such that*

$$\lim_{|x|\to\infty} \frac{|\nabla W(t,x) - V_\infty(t)x|}{|x|} = 0 \quad \text{uniformly for } t \in [0,T]$$

and

$$\inf_{t\in[0,T]} V_\infty(t) > \frac{4\pi^2 + T^2}{T^2} \max\{1, 2b_2\};$$

(W3) $\lim_{|x|\to\infty}[(\nabla W(t,x),x) - 2W(t,x)] = +\infty$ *uniformly for* $t \in [0,T]$.

Then system (1.1) has a nontrivial T-periodic solution.

In view of the proof of [19, Theorem 1.1] (see the proof [19, Lemma 3.1]), the condition (W3) should be stated as in Theorem A instead of as in [19, Theorem 1.1]. Motivated by papers [10,19], in this paper, we will further study the existence of T-periodic solutions of (1.1) under more general conditions. Our first result is the following theorem.

Theorem 1.1. *Assume that F satisfies* (F1), *and that K and W satisfy*

(K1′) *There exist constants $b > 0$ and $\gamma \in (1,2]$ such that*

$$K(t,0) = 0, \qquad K(t,x) \geq b|x|^\gamma \quad \text{for } (t,x) \in [0,T] \times \mathbb{R}^N;$$

(K2′) $(\nabla K(t,x),x) \leq 2K(t,x)$ *for* $(t,x) \in [0,T] \times \mathbb{R}^N$;

(W1′) $\limsup_{|x|\to 0} \frac{W(t,x)}{|x|^2} < b$ *uniformly for* $t \in [0,T]$;

(W3′) *There exists a function $g \in L^1([0,T],\mathbb{R})$ such that*

$$(\nabla W(t,x),x) - 2W(t,x) \geq g(t) \quad \text{for } (t,x) \in [0,T] \times \mathbb{R}^N$$

and

$$\lim_{|x|\to\infty}[(\nabla W(t,x),x) - 2W(t,x)] = +\infty \quad \text{for a.e. } t \in [0,T];$$

(W4) *There exist constants $a > 0$ and $d > 0$ such that*

$$W(t,x) \leq a|x|^2 + d \quad \text{for } (t,x) \in [0,T] \times \mathbb{R}^N;$$

(W5) *There exists $x_0 \in \mathbb{R}^N$ such that*

$$\int_0^T \left[K(t,x_0) - W(t,x_0) - \frac{g(t)}{2}\right] \mathrm{d}t < 0.$$

Then system (1.1) has a nontrivial T-periodic solution.

Corollary 1.1. *Assume that F, K and W satisfy* (F1), (K1′), (K2′), (W1′), (W3′) *and*

(W2′) *There exists a function $V_\infty \in L^\infty([0,T],\mathbb{R})$ such that*

$$\lim_{|x|\to\infty} \frac{|\nabla W(t,x) - V_\infty(t)x|}{|x|} = 0 \quad \text{uniformly for } t \in [0,T]$$

and

$$\int_0^T \left[\max_{|x|=1} K(t,x) - \frac{V_\infty(t)}{2}\right] \mathrm{d}t < 0.$$

Then system (1.1) has a nontrivial T-periodic solution.

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