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Species permanence and dynamical behavior analysis of an impulsively controlled ecological system with distributed time delay[☆]

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ABSTRACT

In this paper, on the basis of the theories and methods of ecology and ordinary differential equations, an ecological system with impulsive harvest and distributed time delay is established. By using the theories of impulsive equations, small amplitude perturbation skills and comparison techniques, we get a condition which guarantees the global asymptotical stability of the prey-(x) eradication and predator-(z) eradication periodic solution. Further, the influences of the impulsive perturbation on the inherent oscillation are studied numerically, and shows rich dynamics, such as period-doubling bifurcation, chaotic bands, periodic windows, chaotic crises, etc. Moreover, the computation of the largest Lyapunov exponent shows the chaotic dynamic behavior of the model. Meanwhile, we investigate the qualitative nature of the strange attractor by using Fourier spectra. All of these results may be useful in the study of the dynamic complexity of ecosystems.

1. Introduction

Community models where consumers share resources have drawn much attention since the pioneering work of Holt [1] on apparent competition [2–4]. A detailed mechanistic understanding of apparent competition is important to assess the effects of alien species invasion on native ecosystems [5–7].

Many evolution processes are characterized by the fact that at certain moments of time they experience a change of state abruptly. These processes are subject to short-term perturbations whose duration is negligible in comparison with the duration of the process [1,2]. Consequently, it is natural to assume that these perturbations act instantaneously, that is, in the form of an impulse. It is well known that biological phenomena involving thresholds, bursting rhythm models in medicine and biology, optimal control models in economics, pharmacokinetics and frequency modulated systems do exhibit impulsive effects [8–14]. Thus impulsive differential equations, differential equations involving impulsive effects, appear as a natural description of observed evolution phenomena of several real world problems [15,16]. Moreover, it is well known that time delay is an important factor of mathematical models in ecology. Usually, time delays in those models have two categories: discrete delay and distributed time delay (continuous delay) [10]. For the impulsive model with distributed time

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delay, papers such as [17–19] have even investigated some ecological models with a distributed time delay and impulsive control strategy.

The model can be described using the following differential equations:

$$\begin{cases} \left.\begin{aligned} \frac{dx(t)}{dt} &= r_1x(t)\frac{(k_0-x(t))}{(k_1-x(t))} - a_2x(t)y(t) - \frac{a_1x(t)z(t)}{b_1+x(t)+c_1z(t)}, \\ \frac{dy(t)}{dt} &= dy(t)\int_{-\infty}^{t}F(t-s)x(s)\,ds + (r_2-m_1)y(t) - d_1y^2(t), \\ \frac{dz(t)}{dt} &= \frac{e_1a_1x(t)z(t)}{b_1+x(t)+c_1z(t)} - m_2z(t), \end{aligned}\right\} & t \neq nT, \\ \left.\begin{aligned} \Delta x(t) &= 0, \\ \Delta y(t) &= -\delta_1 y(t), \\ \Delta z(t) &= 0. \end{aligned}\right\} & t = nT, \end{cases} \tag{1.1}$$

where $x(t), y(t), z(t)$ are the densities of one prey and two predators at time t, respectively, $\Delta x(t) = x(t^+) - x(t)$, $\Delta y(t) = y(t^+) - y(t)$, $\Delta z(t) = z(t^+) - z(t)$, r_i $(i = 1, 2)$ are the intrinsic growth rates, a_i $(i = 1, 2)$ are the cropping rates, e_1 denotes the efficiency with which resources are converted to new consumers, r_1k_0 $\left(0 \leq \frac{k_0}{k_1} \leq 1\right)$ is the carrying capacity of the prey, k_1 is the value of limiting resources, b_1 is a saturation constant, c_1 scales the impact of predator interference, d_1 is the intraspecies density dependence coefficient of the predator y, m_i $(i = 1, 2)$ are the mortality rates for each predator, and d denotes the product of the per-capita rate of predation and the rate of converting prey into predator, The function $F(t)$ satisfies $\int_0^{+\infty} F(s)\,ds = 1$ and $F(t) = ae^{(-at)},\ a > 0$. Then T is the period of the impulsive effect, $n \in N$, N is the set of all non-negative integers, δ_1 $(0 \leq \delta_1 \leq 1)$ is the proportion of harvest at fixed moments $t = nT$.

In order to study the system, we can carry out the chain transform $p(t) = \int_{-\infty}^{t} F(t-s)x(s)\,ds$. Since

$$\int_{-\infty}^{t} F(t-s)\,ds = \lim_{A\to-\infty}\int_A^t ae^{-a(t-s)}\,ds = 1,$$

and $\int_{-\infty}^{t} F(t-s)x(s)\,ds$ is convergent, then

$$\Delta p(t) = \int_{-\infty}^{t^+} F(t-s)x(s)\,ds - \int_{-\infty}^{t} F(t-s)x(s)\,ds = 0, \quad t = nT,\ n \in N.$$

Furthermore, the system (1.1) becomes

$$\begin{cases} \left.\begin{aligned} \frac{dx(t)}{dt} &= r_1x(t)\frac{(k_0-x(t))}{(k_1-x(t))} - a_2x(t)y(t) - \frac{a_1x(t)z(t)}{b_1+x(t)+c_1z(t)}, \\ \frac{dy(t)}{dt} &= dp(t)y(t) + (r_2-m_1)y(t) - d_1y^2(t), \\ \frac{dz(t)}{dt} &= \frac{e_1a_1x(t)z(t)}{b_1+x(t)+c_1z(t)} - m_2z(t), \\ \frac{dp(t)}{dt} &= a(x(t)-p(t)), \end{aligned}\right\} & t \neq nT, \\ \left.\begin{aligned} \Delta x(t) &= 0, \\ \Delta y(t) &= -\delta_1 y(t), \\ \Delta z(t) &= 0, \\ \Delta p(t) &= 0. \end{aligned}\right\} & t = nT. \end{cases} \tag{1.2}$$

From the above discussions, we know that the properties of system (1.1) can be obtained by investigating system (1.2), therefore, in the following we will mainly consider system (1.2).

In this paper, we give the condition which guarantees the global asymptotical stability of the prey-(x) eradication and predator-(z) eradication periodic solution. It is proved that the system is permanent, via a method of comparison involving multiple Lyapunov functions. Secondly, by using numerical simulation, we investigate the influence on the inherent oscillation caused by the impulsive perturbations. Finally, the computation of the largest Lyapunov exponent shows the chaotic behavior of the model, and the qualitative nature of the strange attractors is studied by using Fourier spectra.

2. Mathematical analysis

Let $R_+ = [0, \infty)$, $R_+^4 = \{X \in R^4 \mid X \geq 0\}$. Denote as $f = (f_1, f_2, f_3, f_4)^\tau$ the map defined by the right hand of the first, second, third and fourth equation of system (1.2). Let $V : R_+ \times R_+^4 \to R_+$, then V is said to belong to class V_0 if: