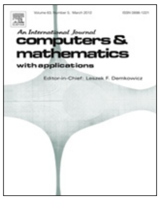
Computers and Mathematics with Applications 69 (2015) 876–892

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa



Efficient and stable model reduction scheme for the numerical simulation of broadband acoustic metamaterials

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HIGHLIGHTS

- The AQSRV scheme is developed for the efficient analysis of the acoustic systems.
- The AQSRV scheme has always a guaranteed convergence of solution.
- The AQSRV scheme automatically constructs the reduced model.
- The heuristic algorithm automatically chooses the subintervals and basis vectors.
- The AQSRV scheme is applied to the broadband acoustic metamaterial systems.

ARTICLE INFO

Article history:
Received 5 June 2014
Received in revised form 6 November 2014
Accepted 1 January 2015
Available online 16 March 2015

Keywords:
Adaptive quasi static Ritz Vector (AQSRV)
Error indicator
Heuristic algorithm for AQSRV
Center frequency
Guaranteed convergence
Acoustic metamaterials

ABSTRACT

This study proposes an efficient and stable model reduction scheme for the numerical simulation of broadband, inhomogeneous, and anisotropic acoustic systems. Unlike a conventional model reduction scheme, the proposed model reduction scheme uses the adaptive quasi-static Ritz vector (AQSRV) as a basis vector. The proposed AQSRV-based model reduction scheme has the following two representative features: (1) Multiple frequency subintervals and (2) Adaptive selection of the subinterval information (i.e., the proper number and location of the center frequencies) and basis vector at each subinterval using the error indicator. "Multiple frequency subintervals" means to divide the frequency band of interest into several frequency bands from the computational time viewpoint. "Adaptive selection of the subinterval information and basis vector" means to select a different number of subintervals and basis vectors for use according to the target system. The proposed model reduction scheme is applied to the numerical simulation of the simple mass–damping–spring system and the acoustic metamaterial systems (i.e., acoustic lens and acoustic cloaking device) for the first time. Through these numerical examples, the proposed model reduction scheme was verified from the efficiency and stability point of view.

1. Introduction

There are many approaches that can be used for the numerical analysis of the dynamic system. Compared with other approaches, the method most widely used is the finite element method (FEM) because FEM can be applied very effectively to complex geometrical shapes and material distributions (i.e., inhomogeneous and anisotropic characteristics) of the system.

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http://dx.doi.org/10.1016/j.camwa.2015.01.004

However, a small finite element is necessary to perform the numerical analysis of the broadband frequency range because the scale of finite elements used is inversely proportional to the frequency of interest. Hence, a very large-scale analysis system (i.e., the large system matrix) and large computer memories are required.

To overcome these limitations of the FEM, the model reduction scheme can be used generally. The main aim of the model reduction scheme is to efficiently compute the approximate response of the system with large-scale and broadband frequency characteristics within a certain error range. Generally, the model reduction scheme can be classified into three approaches. The first approach is based on the domain decomposition [1]. This approach efficiently performs numerical analysis by dividing the target analysis system into several small domains. An iterative solver and parallel computing are used for the numerical analysis. The second approach is based on the power series [2]. The goal of this method is to find appropriate undetermined coefficients by minimizing the least squares error of the power series (i.e., Taylor series, Padé series, etc.) to obtain an approximate response for the broadband frequency range. The third approach is the basis vector-based approach [3–6]. This approach transforms the full system into a more generalized reduced system using the projection matrix, which is composed of the basis vector of the target system. Because the reduced system is used, a very efficient numerical analysis is possible. We focus on the basis vector-based model reduction scheme in this study.

The conventional basis vector-based model reduction schemes have several critical limitations. First, the selection of the appropriate number of basis vectors for each target system is difficult. In general, the optimal number of basis vectors is different for each target system. Namely, this selection process is very dependent on the system and is empirical. Second, in the existing conventional model reduction scheme, the selection of subinterval information (i.e., the number and location of the center frequencies) is difficult. Moreover, this process is performed manually. Thus, even experienced users have difficulty performing this process. This process has the disadvantage of not guaranteeing the stability of the solution (i.e., is the approximate solution always converged?). Therefore, these limitations should be overcome for the efficient broadband analysis and design of the systems in the future.

Meanwhile, acoustic metamaterial systems have received attention from many researchers for the last 10 years. In addition, many studies related to this topic have been performed. Acoustic metamaterials are artificial materials with peculiar acoustic characteristics (e.g., a negative refractive index, negative density, or negative bulk modulus) that are never found in nature. More recently, the meaning of acoustic metamaterials has been extended to all types of systems that control the behavior of an acoustic wave through an artificially designed unit microstructure (e.g., phononic crystals). To date, various application systems have used these acoustic metamaterials. Representative systems include the acoustic lens using a negative refractive index [7], acoustic gradient-index metamaterial-based acoustic lens [8–10], acoustic cloaking device [11], and noise barrier [12]. Most of these studies rely more on intuitive, empirical, and experimental approaches. However, an efficient and stable numerical analysis-based theoretical approach is desired to develop an improved acoustic metamaterial system design.

Thus, in this study, we propose the AQSRV-based model reduction scheme to complement the aforementioned limitations of these conventional basis vector-based model reduction schemes. The AQSRV-based model reduction scheme has two characteristics: (1) Multiple frequency subintervals and (2) Adaptive selection of the subintervals and basis vectors. Furthermore, this study applies the proposed AQSRV-based model reduction scheme to the acoustic metamaterial systems and verifies its performance. The acoustic metamaterial systems are chosen as the validation example for two reasons. First, many researchers are interested in the field of acoustic metamaterials. In addition, the acoustic metamaterial systems have the broadband characteristic. Thus, it is a suitable application for the proposed method.

In this respect, the main contributions of this study are classified into two parts: First, we propose the AQSRV-based model reduction scheme that can automatically select both the optimal number of subintervals and the basis vectors for the target system using an error indicator. Additionally, we propose the efficient heuristic algorithm for this process. Using this approach, the computational efficiency and the accuracy of the approximate response are simultaneously satisfied. Moreover, the proposed method always has a guaranteed convergence of solution and provides the convenience of automatically constructing the reduced model without experiences for the target system. Second, the model reduction scheme is applied for the first time for the numerical analysis of inhomogeneous and anisotropic acoustic metamaterial systems. Note that in addition to the acoustic metamaterial system, the present proposed method can be used for nearly all systems that have a broadband dynamic behavior (e.g., NVH, acoustics, electromagnetics, etc.).

2. Modeling of the inhomogeneous and anisotropic acoustic systems

In this section, the governing equation, boundary conditions, and finite element model are defined before performing the numerical analysis of the acoustic systems. This finite element model is used in the algorithm to obtain the basis vectors of the proposed AQSRV-based model reduction scheme. A more detailed description for this algorithm will be provided in Section 3.

2.1. Governing equation, boundary conditions, and finite element model

In general, acoustic metamaterials have inhomogeneous and anisotropic material characteristics (e.g., mass density tensor $\boldsymbol{\rho}$ and bulk modulus κ). Thus, we apply the inhomogeneous and anisotropic Helmholtz equation as the governing equation to model the acoustic metamaterial system in this study. The Helmholtz equation is represented as Eq. (1). Then,