Computers and Mathematics with Applications 59 (2010) 2703–2710

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Four positive periodic solutions to two species parasitical system with harvesting terms[☆]

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ARTICLE INFO

Article history:
Received 16 June 2009
Received in revised form 15 January 2010
Accepted 19 January 2010

Keywords:
Four positive periodic solutions
Parasitical system
Coincidence degree
Harvesting term

ABSTRACT

By using Mawhin's continuation theorem of coincidence degree theory, we establish the existence of four positive periodic solutions for two species parasitical system with harvesting terms. An example is given to illustrate the effectiveness of our results.

1. Introduction

In recent years, the existence of periodic solutions in biological models has been widely studied. Models with harvesting terms are often considered. Generally, the model with harvesting terms is described as follows:

$$\dot{x} = xf(x, y) - h, \qquad \dot{y} = yg(x, y) - k,$$

where x and y are functions of two species, respectively; h and k are harvesting terms standing for the harvests (see [1,2]). Because of the effect of changing environment such as the weather, season, food and so on, the number of species population periodically varies with the time. The rate of change usually is not a constant. Motivated by this, we consider the periodic non-autonomous population models. In this paper, we consider the following two species parasitical system with harvesting terms:

$$\begin{cases} \dot{x} = x(t)(a_1(t) - b_1(t)x(t)) - h_1(t), \\ \dot{y} = y(t)(a_2(t) - b_2(t)y(t) + c(t)x(t)) - h_2(t), \end{cases} \tag{1.1}$$

where, $x(t)$ and $y(t)$ denote the densities of the host and the parasites, respectively; $a_i(t)$, $b_i(t)$, $c(t)$ and $h_i(t)$ $(i = 1, 2)$ are all positive continuous functions and denote the intrinsic growth rate, death rate, obtaining nutriment rate from the host, harvesting rate, respectively. In model (1.1), the parasitical influence on its host is negligible. The parasitical phenomenon described by model (1.1) is universal in the biological system or ecosystem. For example, holding within limits, the ascarid does no harm to the people. Of course, the people live very well without the ascarid. On the existence of positive periodic solutions to systems (1.1), few results are found in the literature. This motivates us to investigate the existence of a positive periodic or multiple positive periodic solutions for system (1.1). In fact, it is more likely for some biological species to take

☆ This work is supported by the National Natural Sciences Foundation of People's Republic of China under Grant 10971183.

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0898-1221/$ – see front matter
doi:10.1016/j.camwa.2010.01.038

on multiple periodic change regulations and have multiple local stable periodic phenomena. Therefore it is essential to for us to investigate the existence of multiple positive periodic solutions for population models. Recently, the powerful and effective approaches such as method of coincidence degree, Lyapunov functional method and differential inequality techniques have been applied to study the existence of periodic or almost periodic solutions in periodic or almost periodic systems. A number of good results have been obtained (see [3–16]). However, the existence of multiple periodic solutions established by using coincidence degree theory for periodic systems are very scare (see [17]). So, in this paper, our purpose is to study the existence of multiple positive periodic solutions to system (1.1). Therefore, we assume that all parameters in system (1.1) are positive ω-periodic functions with $\omega > 0$.

The organization of the rest of this paper is as follows. In Section 2, by employing the continuation theorem of coincidence degree theory, we establish the existence of four positive periodic solutions of system (1.1). In Section 3, an example is given to illustrate the effectiveness of our results.

2. Existence of four positive periodic solutions

For convenience, we introduce some concepts from the book by Gains and Mawhin.

Let X and Z be Banach spaces. A linear mapping $L : \operatorname{Dom} L \subset X \to Z$ is called Fredholm if its kernel $\operatorname{Ker} L = \{X \in \operatorname{Dom} L : Lx = 0\}$ has finite dimension and its range $\operatorname{Im} L = \{Lx : x \in \operatorname{Dom} L\}$ is closed and has finite codimension. The index of L is defined by the integer $\dim \operatorname{Ker} L - \operatorname{codim} \operatorname{Im} L$. If L is a Fredholm mapping with index zero, then there exists continuous projections $P : X \to X$ and $Q : Z \to Z$ such that $\operatorname{Im} P = \operatorname{Ker} L$ and $\operatorname{Ker} Q = \operatorname{Im} L$. Then $L|_{\operatorname{Dom} L \cap \operatorname{Ker} P} : \operatorname{Im} L \cap \operatorname{Ker} P \to \operatorname{Im} L$ is bijective, and its inverse mapping is denoted by $K_P : \operatorname{Im} L \to \operatorname{Dom} L \cap \operatorname{Ker} P$. Since $\operatorname{Ker} L$ is isomorphic to $\operatorname{Im} Q$, there exists a bijection $J : \operatorname{Ker} L \to \operatorname{Im} Q$. Let Ω be a bounded open subset of X and let $N : X \to Z$ be a continuous mapping. If $QN(\overline{\Omega})$ is bounded and $K_P(I - Q)N : \overline{\Omega} \to X$ is compact, then N is called L-compact on Ω, where I is the identity.

Let L be a Fredholm linear mapping with index zero and let N be a L-compact mapping on $\overline{\Omega}$. Define mapping $F : \operatorname{Dom} L \cap \overline{\Omega} \to Z$ by $F = L - N$. If $Lx \neq Nx$ for all $x \in \operatorname{Dom} L \cap \partial\Omega$, then by using P, Q, K_P, J defined above, the coincidence degree of F in Ω with respect to L is defined by

$$\operatorname{Deg}_L(F, \Omega) = \deg(I - P - (J^{-1}Q + K_P(I - Q))N, \Omega, 0),$$

where $\deg(g, \Gamma, p)$ is the Leray–Schauder degree of g at p relative to Γ.

Then the Mawhin's continuous theorem is given as follows:

Lemma 2.1 ([18]). *Let $\Omega \subset X$ be an open bounded set and let $N : X \to Z$ be a continuous operator which is L-compact on $\overline{\Omega}$. Assume*

(a) *for each $\lambda \in (0, 1)$, $x \in \partial\Omega \cap \operatorname{Dom} L$, $Lx \neq \lambda Nx$;*
(b) *for each $x \in \partial\Omega \cap L$, $QNx \neq 0$;*
(c) *$\deg(JNQ, \Omega \cap \operatorname{Ker} L, 0) \neq 0$.*

Then $Lx = Nx$ has at least one solution in $\overline{\Omega} \cap \operatorname{Dom} L$.

For the sake of convenience, we introduce some notations

$$f^l = \min_{t\in[0,\omega]} f(t), \qquad f^M = \max_{t\in[0,\omega]} f(t), \qquad \bar{f} = \frac{1}{\omega}\int_0^{\omega} f(t)\,\mathrm{d}t,$$

here $f(t)$ is a continuous ω-periodic function.

Throughout this paper, we need the following assumptions.

(H_1) $a_1^l > 2\sqrt{b_1^M h_1^M}$;
(H_2) $a_2^l > 2\sqrt{b_2^M h_2^M}$;
(H_3) $c^M l_1^+ > \sqrt{(a_2^l)^2 - 4b_2^M h_2^M}$.

For simplicity, we also introduce five positive numbers as follows.

$$l_1^{\pm} = \frac{a_1^M \pm \sqrt{(a_1^M)^2 - 4b_1^l h_1^l}}{2b_1^l}, \qquad l_2^{\pm} = \frac{a_2^l \pm \sqrt{(a_2^l)^2 - 4b_2^M h_2^M}}{2b_2^M}, \qquad K = \frac{a_1^M b_1^M}{a_1^l b_1^l}.$$

Lemma 2.2. *For the following equation*

$$a_1(t) - b_1(t)\mathrm{e}^{u_*(t)} - h_1(t)\mathrm{e}^{-u_*(t)} = 0, \quad \textit{for all } t \in R,$$

if assumption (H_1) holds, then we have the following inequality

$$\ln l_1^- < u_*^- < \ln\left(\frac{l_1^+ + l_1^-}{2K}\right) < u_*^+ < \ln l_1^+, \quad \textit{for all } t \in R,$$