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Extended precise integration method for consolidation of transversely isotropic poroelastic layered media

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ABSTRACT

A new numerical method for calculating the consolidation behavior of the stratified, transversely isotropic and poroelastic material is presented by combining the extended precise integration algorithm with the integral transformation techniques. Starting with the governing partial differential equations of a saturated medium with transversely isotropic skeleton and compressible fluid constituents, an ordinary differential matrix equation is deduced with the aid of a Laplace–Hankel transform. An extended precise integration method for internal loading situations is proposed to solve the ordinary differential matrix equation in the transformed domain, and the actual solution is recovered by a numerical inverse transformation. Numerical examples are also provided to prove the feasibility of this method.

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1. Introduction

In engineering analysis and design, many materials can be regarded as saturated, poroelastic media consisting of solid grain and pore fluid, and their mechanical response has also gained wide attention from researchers and engineers. Owing to Biot's pioneering work [1,2] on fluid-saturated porous solid, researchers and engineers may now investigate the time-dependent behavior of saturated media based on a more reasonable theory. However, differing from the homogeneous assumption for a half space or a single layer, actual materials may possess layered structures, especially for sedimented materials, such as natural soil profiles. For multilayered and saturated media, numerous advisable approaches have been developed by combining with Biot's consolidation theory, including analytical methods [3–9], semi-analytical methods [10–14], and numerical methods [15–20].

Booker and Small [15,16] firstly proposed a finite layer solution for consolidation by utilizing the integral transformation and numerical approximation, and then derived a more efficient and direct Laplace inversion approach [18] to avoid the time-marching algorithm. Vardoulakis and Harnpattanapanich [17,19] presented a similar method in their works to tackle the layered soil consolidation problem. Mei et al. [11] applied a varied finite layer method to calculate the consolidation of transversely isotropic soils, in which the integral transform to the horizontal coordinate variables was replaced by a series expansion, thus the accuracy and efficiency of the results were strongly dependent on the truncation of the series. In order to improve numerical stability and precision, both of the exact stiffness matrix method [10] and the analytical layer-element method [12–14] were put forward based on a similar methodology.

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The state space method (or called transfer matrix method) is also widely applied to calculate the consolidation of stratificated media. Pan [3] presented the complete Green's functions in a multilayered half-space with the aid of propagator matrices. Wang and Fang [4,5] employed the state space approach to analyze the axisymmetric and non-axisymmetric consolidation of multilayered media. Ai et al. [7,8] took advantage of the transfer matrix method to investigate the consolidation problems in cylindrical and Cartesian coordinate systems. In other researches [6,9], the anisotropic permeability of solid skeleton and the compressibility of pore fluid were also taken into account.

Compared to the methods mentioned above, the traditional numerical methods which contain the finite element method [21,22], the boundary element method [20,23] as well as the finite difference method [24] are more powerful in dealing with complex boundary or loading conditions, and they are proved theoretically feasible to solve consolidation problems of multilayered poroelastic media. With the development of commercial softwares, more and more designers tend to use them in analyzing related problems. However, it has to be pointed out that their implementations are still time-consuming and expensive. In addition, for the fully discrete mesh-based methods, their computational accuracy and convergence in a consolidation problem may be affected by many factors, such as the model dimension, the meshing size, and the initial time step. Therefore, they may not be the best choice for the preliminary design in engineering.

Based on the transfer matrix method, the first author and his cooperators deduced the systematic consolidation solutions for multilayered structures in different coordinate systems [7,8]. However, in most analytical methods including the transfer matrix method, the existence of positive exponential functions may create numerical instability, which consequently hinders their application and development. In order to eliminate this influence in numerical calculation, the analytical layer-element method [12–14] was further developed to calculate the consolidating soils. Moreover, the negative exponential functions of provided elements of the analytical layer-element could avoid the overflow in computation. It has to be indicated that the preceding analytical solutions were proposed especially for the consolidation of isotropic multilayered media, and it would be an arduous task to get the explicit solutions for more complex models, such as the transversely isotropic model.

Besides the foregoing discussion, it is still significant to lay emphasis on the efficiency and stability in consolidation calculation. This paper aims to present an alternative numerical method for the calculation of multilayered media and apply it to the analysis of the consolidation of poroelastic materials. Considering the main engineering properties, the author assumed the physical system to be a transversely isotropic poroelastic medium filled with compressible pore fluid. With the aid of Laplace–Hankel transform, the original governing equations in the cylindrical coordinate system are deduced to the standard differential matrix equation in the transformed domain, which reduces the partial differential equations to ordinary differential ones. Since the extension of the precise integration method (PIM) [25] has been proved successfully applicable in analyzing wave propagation problems in layered media [26–28], a calculation process for quasi-static analysis of multilayered systems with internal force sources is presented and then applied to solve the ordinary differential matrix equations. With the inherent advantages of PIM, the present method can avoid exponential overflow in numerical calculation and its efficiency and precision can be guaranteed by adopting optimized numerical inversion approaches. Numerical examples are given to discuss the efficiency and accuracy of the present method, and one more example is designed to illustrate the influence of stratification of material on the consolidation behavior.

2. The matrix differential equation of Biot's consolidation

2.1. Fundamental governing partial differential equations

It is worth mentioning that the studied subject in this paper is mainly about the consolidation performance of the materials within the framework of small deformation such as the normally consolidated soil, of which the self-weight consolidation has finished and the body force can be ignored.

The partial differential equations of equilibrium without body force in the cylindrical coordinate system are given by

$$\frac{\partial \sigma_r}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{r\theta}}{\partial \theta} + \frac{\partial \sigma_{rz}}{\partial z} + \frac{\sigma_r - \sigma_\theta}{r} = 0, \tag{1a}$$

$$\frac{\partial \sigma_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\theta}}{\partial \theta} + \frac{\partial \sigma_{\theta z}}{\partial z} + \frac{2\sigma_{r\theta}}{r} = 0, \tag{1b}$$

$$\frac{\partial \sigma_{rz}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\theta z}}{\partial \theta} + \frac{\partial \sigma_{z}}{\partial z} + \frac{\sigma_{rz}}{r} = 0, \tag{1c}$$

where $\sigma_r, \sigma_\theta, \sigma_z$ are normal stress components in the r, θ, and z directions, respectively; $\sigma_{rz}, \sigma_{z\theta}, \sigma_{r\theta}$ are shear stress components in the $r - z, \theta - z$, and $\theta - r$ planes, respectively.

The principle of effective stress can be expressed as

$$\boldsymbol{\sigma}_s = \boldsymbol{\sigma}'_s - \boldsymbol{\sigma}, \tag{2}$$

where $\boldsymbol{\sigma}_s = [\sigma_r, \sigma_\theta, \sigma_z, \sigma_{rz}, \sigma_{\theta z}, \sigma_{r\theta}]^{\mathrm{T}}$ and $\boldsymbol{\sigma}'_s = \left[\sigma'_r, \sigma'_\theta, \sigma'_z, \sigma'_{rz}, \sigma'_{\theta z}, \sigma'_{r\theta}\right]^{\mathrm{T}}$ are the vectors of total stresses and effective stresses, respectively; $\boldsymbol{\sigma} = [\sigma, \sigma, \sigma, 0, 0, 0]^{\mathrm{T}}$ is the excess pore fluid pressure vector (pressure as positive).

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