Computers and Mathematics with Applications 68 (2014) 1844–1870

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Artificial boundary conditions for axisymmetric eddy current probe problems

Houssem Haddar [a], Zixian Jiang [a,*], Armin Lechleiter [b]

[a] INRIA Saclay – Ecole Polytechnique, 91120 Palaiseau, France
[b] University of Bremen, 28359 Bremen, Germany

ARTICLE INFO

Article history:
Received 28 October 2013
Received in revised form 4 August 2014
Accepted 5 October 2014
Available online 25 October 2014

Keywords:
Axisymmetric eddy current equations
Artificial boundary conditions
Non-selfadjoint spectral problem
DtN maps

ABSTRACT

We study different strategies for the truncation of computational domains in the simulation of eddy current probes of elongated axisymmetric tubes. For axial fictitious boundaries, an exact Dirichlet-to-Neumann map is proposed and mathematically analyzed via a non-selfadjoint spectral problem: under general assumptions we show convergence of the solution to an eddy current problem involving a truncated Dirichlet-to-Neumann map to the solution on the entire, unbounded axisymmetric domain as the truncation parameter tends to infinity. Under stronger assumptions on the physical parameters of the eddy current problem, convergence rates are shown. We further validate our theoretical results through numerical experiments for a realistic physical setting inspired by eddy current probes of nuclear reactor core tubes.

1. Introduction

1.1. Industrial context

The present work is motivated by eddy current non-destructive testing of steam generators (SG), see Fig. 1, which are critical components in nuclear power plants. Heat produced in a nuclear reactor core is transferred into the primary loop of a steam generator, consisting of tubes in U-shape, and boils coolant water in the secondary loop on the shell side into steam. This steam is then delivered to turbines generating electrical power. Conductive magnetic deposits usually observed on the shell side of the U-tubes could, however, affect the power production and even the structure security. The upper part of the tubes of U-shape is accessible to normal inspection from the top of the steam generator. But it is difficult to reach the lower part of the tubes, which are straight and long, without disassembling the SG. Therefore, a non-destructive examination procedure using signals of eddy currents is applied to detect the presence, the shape and/or the physical nature of deposits on the lower part of the U-tubes.

In eddy current testing (ECT), we use a probe consisting of two coils of wire. Each of these coils is connected to a current generator producing an alternating current and to a voltmeter measuring the voltage change across the coil. Once the probe is introduced in the lower part of some U-tube, the generator coil excited by the current creates a primary electromagnetic field which in turn induces a current flow in the electrically conductive material nearby, such as the tube itself. The presence of conductive magnetic deposits will distort the flow of the eddy currents. They induce a current change in the receiver coil which is measured in terms of impedance and is called the ECT signal (cf. [1,2]).

* Corresponding author.
E-mail addresses: houssem.haddar@inria.fr (H. Haddar), zixian.jiang@polytechnique.edu (Z. Jiang), lechleiter@math.uni-bremen.de (A. Lechleiter).

http://dx.doi.org/10.1016/j.camwa.2014.10.008
0898-1221/

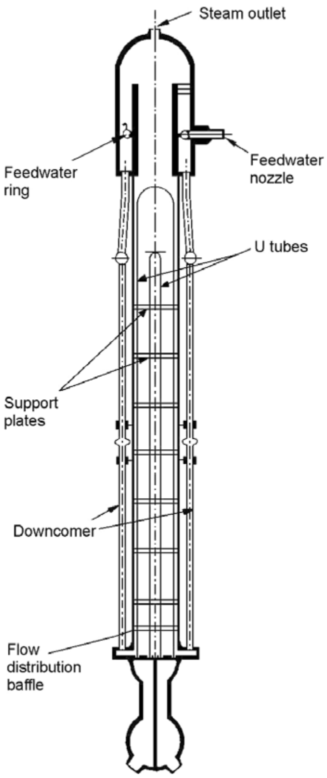

Fig. 1. Sketch of a steam generator.

In order to simulate an eddy current testing experiment, one needs to solve the forward problem for any probe position one wants to incorporate into the measurements. For an iterative inversion method based on the exploitation of this forward problem, the number of required simulation is also proportional to the number of iterations. Given the large number of tubes to be probed, one easily understands the crucial importance of designing a fast (and reliable) numerical simulation of the forward problem. We consider here the eddy current problem under axisymmetric assumption (see for instance [3]) and investigate strategies to bound the computational domain. While for the radial direction, truncation with brute model for the boundary condition such as Neumann boundary condition would be sufficient due to the conductivity of the tube and the decay of the solution, in the axial direction this strategy requires some fictitious boundaries far from the sources. We rather propose to compute the exact Dirichlet-to-Neumann (DtN) operator for the region outside the source term and apply it as an exact boundary condition on the fictitious boundaries. This would allow the latter to be as close as needed to the source term. DtN boundary conditions for domain cut-off are widely studied in waveguides and gratings [4–8]. The main difficulty here is in the justification of analytical expansion using spectral decomposition for the DtN map and, for numerical reasons, in the error estimate for the truncated expansion of DtN map. We shall rely on results from perturbation theory for the spectrum of compactly perturbed selfadjoint operators. We also study the error due to truncation in the expression of the DtN operator and relate this to the regularity of the problem parameters. Indeed the latter is important from the computational point of view since this truncation is needed in practice. The DtN expansion relies on some eigenvalues and eigenfunctions that are not known analytically and should be numerically approximated. This may be expensive if a high degree of precision is required. However these calculations can be done off-line and therefore would not affect the speed of solving the problem.

There is a large literature on eddy current problems and without being exhaustive we may refer to [9] for a recent survey on the problem, including an introduction to the eddy current phenomenon, the mathematical justification of the eddy current approximation and different formulations and numerical approaches for the three-dimensional problems. For axisymmetric configurations we refer to the work of [10] for the study of the theoretic tools for the Maxwell equations in three dimensions, and to the works of [3,11] for the discussion of the eddy current problem with bounded conductive components in the meridian half-plane, the numerical analysis and some numerical experiments applied to the induction heating system.

The paper is organized as follows. In Section 2, we briefly recall the eddy current model in the cylindrical coordinate system corresponding to the rotational symmetry with respect to the axis of the tube (see Fig. 2) and discuss existence and uniqueness of solution to this problem in its equivalent variational formulation in properly defined weighted function spaces.

We then introduce truncations of the domain in the radial-direction by introducing some local boundary conditions (see Section 2.1) and then in the axial-direction by constructing the DtN boundary operator (see Section 3). We validate our