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The effect of thermal radiation on the flow of a second grade fluid

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ABSTRACT

This paper reports the magnetohydrodynamic (MHD) flow and heat transfer characteristics of a second grade fluid in a channel. Analytic technique namely the homotopy analysis method (HAM) is used to solve the momentum and energy equations. The important findings in this paper are the effects of second grade parameter, Hartmann number, Reynolds number, thermal radiation parameter, Prandtl and local Eckert numbers on the velocity, temperature, skin friction coefficient and Nusselt number.

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1. Introduction

The flow behavior of rheological fluids is a topic of current interest because of its engineering and industrial importance; for instance processing of polymers, biomechanics, enhanced oil recovery and food products etc. Different from the viscous fluids, it is impossible to characterize all the rheological fluids by a single constitutive equation. Therefore several constitutive equations of such fluids have been reported in the literature. Amongst these there is the simplest one which is known as the second grade fluid. In view of its simplicity, various investigators [1–10] in the field have recently used it in different flow descriptions. It is noticed that the second grade fluids can predict the normal stress effects. However, such fluids do not exhibit the shear thinning/thickening effects [11–13]. These fluids also do not possess the characteristics of relaxation and retardation phenomena.

The influence of complex rheological parameters on the heat transfer is a topic of great interest to the researchers nowadays. Such analysis has potential applications in extrusion process, hemodialysis and oxygenation. In the light of such motivations, the present work studies the effects of thermal radiation [14] on the MHD flow in a channel with stretching walls. Constitutive equations of second grade fluid are considered. Series solutions for velocity and temperature are constructed by homotopy analysis method [15,16]. This method has been already used by different authors for many problems [17–30]. Here the convergence of the derived solutions is ensured. The variation of emerging parameters are discussed on the flow quantities of interest.

2. Development of the flow problem

Let us consider the two-dimensional and steady flow of a second grade fluid in a channel bounded by the planes $y = \pm a$. The x- and y-axes are chosen parallel to the channel walls and perpendicular to the flow respectively. A constant magnetic

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field $\mathbf{B}_0$ is applied in a direction transverse to the flow. Symmetry in the flow is taken into account about the line $y = 0$. The flow is generated due to stretching of the channel walls. The heat transfer in the channel is because of the constant temperature to the channel walls. In addition heat radiation effects are included. Constitutive expressions in a second grade fluid are

$$\mathbf{T} = -p\mathbf{I} + \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2, \tag{1}$$

$$\mathbf{A}_1 = \nabla\mathbf{V} + (\nabla\mathbf{V})^{\mathrm{T}}, \tag{2}$$

$$\mathbf{A}_2 = \frac{\mathrm{d}\mathbf{A}_1}{\mathrm{d}t} + \mathbf{A}_1(\nabla\mathbf{V}) + (\nabla\mathbf{V})^{\mathrm{T}}\mathbf{A}_1, \tag{3}$$

in which p is pressure, $\mathbf{I}$ is unit tensor, $\mathbf{V}$ is the velocity and $\mathrm{d}/\mathrm{d}t$, μ, α_1, α_2 are material derivative, dynamic viscosity, viscoelasticity, cross viscosity respectively. Further more μ, α_1 and α_2 satisfy [13]

$$\mu \geq 0, \qquad \alpha_1 \geq 0, \qquad \alpha_1 + \alpha_2 = 0. \tag{4}$$

The relevant boundary layer problems are

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u + \frac{\alpha_1}{\rho}\begin{bmatrix} u\dfrac{\partial^3 u}{\partial x\partial y^2} + v\dfrac{\partial^3 u}{\partial y^3} \\ +\dfrac{\partial u}{\partial x}\dfrac{\partial^2 u}{\partial y^2} - \dfrac{\partial u}{\partial y}\dfrac{\partial^2 u}{\partial x\partial y}\end{bmatrix}, \tag{5}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{c_p\rho}\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 - \frac{1}{c_p\rho}\frac{\partial q_r}{\partial y} + \frac{\alpha_1}{\rho c_p}\begin{bmatrix} u\dfrac{\partial u}{\partial y}\dfrac{\partial^2 u}{\partial x\partial y} + v\dfrac{\partial u}{\partial y}\dfrac{\partial^2 u}{\partial y^2} \\ +\dfrac{\partial v}{\partial y}\left(\dfrac{\partial u}{\partial y}\right)^2 + \dfrac{\partial u}{\partial x}\left(\dfrac{\partial u}{\partial y}\right)^2\end{bmatrix}, \tag{6}$$

$$\begin{aligned} &u = bx, \qquad v = 0 \quad \text{at } y = a; \quad b > 0, \\ &\frac{\partial u}{\partial y} = 0 \qquad v = 0 \quad \text{at } y = 0, \\ &T = T_w, \quad \text{at } y = a, \\ &\frac{\partial T}{\partial y} = 0 \quad \text{at } y = 0, \end{aligned} \tag{7}$$

where $u, v, T, T_w, \nu, \sigma, \rho, k, c_p$ and q_r are components of velocity in x and y directions, temperature, the wall temperature, kinematic viscosity, electrical conductivity, mass density, thermal conductivity, specific heat and heat flux respectively. All the fluid properties are taken constant. By the Rosseland approximation the radiative heat flux can be reduced in the form

$$q_r = -\frac{4\Gamma}{3k^*}\frac{\partial T^4}{\partial y}, \tag{8}$$

where Γ and k^* are the Stefan–Boltzmann constant and the mean absorption coefficient respectively. Invoking Taylor series, one has

$$T^4 \approx 4T_0^3 T - 3T_0^4, \tag{9}$$

where in the above equation T_0 is the temperature at the central line $y = 0$ and terms of higher order are neglected.

Employing the following transformations

$$u = bxf'(\eta), \qquad v = -abf(\eta), \qquad \eta = \frac{y}{a}, \qquad \theta = \frac{T}{T_w}, \tag{10}$$

we have

$$f''' - \mathrm{Re}\left(\left(f'\right)^2 - ff''\right) - M^2 f' + \alpha\begin{bmatrix} 2f'f''' - ff^{(iv)} \\ -\left(f''\right)^2 \end{bmatrix} = 0, \tag{11}$$

$$f(0) = 0, \qquad f(1) = 0, \qquad f'(1) = 1, \qquad f''(0) = 0, \tag{12}$$

$$\left(\frac{3N_R + 4}{3N_R}\right)\theta'' + \Pr\mathrm{Re} f\theta' + \Pr Ec\left(f''\right)^2 + \alpha\Pr Ec\begin{pmatrix} f'f''^2 \\ -ff''f''' \end{pmatrix} = 0, \tag{13}$$

$$\theta'(0) = 0, \qquad \theta(1) = 1, \tag{14}$$

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