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Population dynamics in presence of state dependent fluctuations

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ABSTRACT

We discuss a model of a system of interacting populations for the case when: (i) the growth rates and the coefficients of interaction among the populations depend on the populations densities; and (ii) the environment influences the growth rates and this influence can be modelled by a Gaussian white noise. The system of model equations for this case is a system of stochastic differential equations with: (i) deterministic part in the form of polynomial nonlinearities; and (ii) state-dependent stochastic part in the form of multiplicative Gaussian white noise. We discuss both the cases when the formal integration of the stochastic differential equations leads: (i) to integrals of Itô kind; or (ii) to integrals of Stratonovich kind. The systems of stochastic differential equations are reduced to the corresponding Fokker–Planck equations. For the Itô case and for the case of 1 population analytic results are obtained for the stationary p.d.f. of the population density. For the case of more than one population and for both the Itô case and Stratonovich case the detailed balance conditions are not satisfied. As a result the exact analytic solutions of the corresponding Fokker–Planck equations for the stationary p.d.f.s for the population densities are not known. We obtain approximate solutions for this case by the method of adiabatic elimination.

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1. Introduction

The research on the nonlinear dynamics of the complex systems increases steadily in the last two decades (see Appendix A for the literature). Many complex systems are influenced by random events. Because of this the theory of stochastic processes is more used for modelling of the processes in the complex systems [1–5]. In this paper we discuss some mathematical aspects of the theory of interacting populations for the case when the growth rates are influenced by environmental fluctuations. For the case when the fluctuations can be modelled by Gaussian white noise we shall obtain as model equations a system of stochastic differential equations that contain multiplicative noise. For the case of single population the model equation will be of the kind

$$\dot{\rho} = F(\rho) + \eta G(\rho) \tag{1.1}$$

where $F(\rho)$ and $G(\rho)$ are polynomials of ρ and η is Gaussian white noise whereas for the system of interacting populations the corresponding model equations will be of the kind

$$\dot{\rho}_i = F_i(\rho_1, \dots, \rho_n) + \eta_i G_i(\rho_1, \dots, \rho_n); \quad i = 1, \dots, n \tag{1.2}$$

where F and G are polynomials of $\rho_1 \dots, \rho_n$ and η_i are Gaussian white noises.

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The organization of the article is as follows. We discuss the model equations for the dynamics of interacting populations in the following section. The presence of Gaussian white noise in the growth rates of populations leads to a system of stochastic differential equations with multiplicative noise. The integration of these stochastic differential equations leads in principle to stochastic integrals of Itô kind or to stochastic integrals of Stratonovich kind. Section 3 is devoted to the theory for the case when the stochastic integrals are of Itô kind. Section 4 is devoted to theory for the case when the stochastic integrals are of Stratonovich kind. Several concluding remarks are summarized in Section 5. In addition the four appendices presented at the end supply the reader with information about the examples of research on complex systems, about the theory of stochastic differential equations containing multiplicative white noise, theory of stochastic differential equations of Itô and Stratonovich kind and their relation to the Fokker–Planck equation (known also as the forward Kolmogorov equation).

2. Investigated equations and population dynamics

The classical model of interacting populations is based on a system of equations of the Lotka–Volterra kind [6,7]:

$$\dot{\rho}_i = r_i \rho_i(t) \left(1 - \sum_{j=1}^{n} \alpha_{ij} \rho_j(t)\right) \tag{2.1}$$

where ρ_i are the densities of the population members, r_i are the birth rates, and α_{ij} are coefficients of interaction between the populations i and j. Suppose that the birth rates and interaction coefficients depend on the density of the populations and the birth rates fluctuate as follows:

$$r_i = r_i^0 \left(1 + \sum_{j=1}^{n} r_{ij} \rho_j\right) + \eta_i; \qquad \alpha_{ij} = \alpha_{ij}^0 \left(1 + \sum_{j=1}^{n} \alpha_{ijk} \rho_k\right) \tag{2.2}$$

where r_i^0 and α_{ij}^0 are the values of the growth rates and interaction coefficients in absence of density dependence, r_{ij} and α_{ijk} are parameters, and η_i are Gaussian white noises. The system of equations (2.2) for $\eta_i = 0$ has been introduced and investigated in [8,9]. The presence of η_i however influences the system dynamics [10,11].

The substitution of Eq. (2.2) in Eq. (2.1) leads to a system of model equations of the kind

$$\begin{aligned}
&\dot{\rho}_i = F_i(\rho_1, \dots, \rho_n) + \eta_i G_i(\rho_1, \dots, \rho_n);\\
&F_i(\rho_1, \dots, \rho_n) = r_i^0 \rho_i \left\{1 - \sum_{j=1}^{n} (\alpha_{ij}^0 - r_{ij}) \rho_j - \sum_{j=1}^{n} \sum_{l=1}^{n} \alpha_{ij}^0 (\alpha_{ijl} + r_{il}) \rho_j \rho_l - \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{l=1}^{n} \alpha_{ij}^0 r_{ik} \alpha_{ijl} \rho_j \rho_k \rho_l\right\}\\
&G_i(\rho_1, \dots, \rho_n) = \rho_i \left(1 - \sum_{j=1}^{n} \alpha_{ij}^0 \rho_j - \sum_{j=1}^{n} \sum_{k=1}^{n} \alpha_{ij}^0 \alpha_{ijk} \rho_j \rho_k\right).
\end{aligned} \tag{2.3}$$

For the case of one population (we set $r^0 = r$; $r_{11} = 0$; $\alpha_{11}^0 = \alpha$; $\alpha_{111} = 0$) the model equation is given by

$$\begin{aligned}
&\dot{\rho} = F(\rho) + \eta G(\rho)\\
&F(\rho) = r\rho - \alpha r \rho^2; \qquad G(\rho) = \rho - \alpha \rho^2.
\end{aligned} \tag{2.4}$$

Below we shall discuss the more general equation in comparison to Eq. (2.4). We shall discuss the case where $F(\rho)$ and $G(\rho)$ are polynomials of arbitrary orders p_1 and p_2, i.e.,

$$F(\rho) = \sum_{i=1}^{p_1} \mu_i \rho^i; \qquad G(\rho) = \sum_{i=1}^{p_2} \theta_i \rho^i \tag{2.5}$$

where μ_i and θ_i are parameters. In this case Eq. (2.4) becomes

$$\dot{\rho} = \sum_{i=1}^{p_1} \mu_i \rho^i + \eta \sum_{i=1}^{p_2} \theta_i \rho^i. \tag{2.6}$$

The formal integration of Eq. (2.4) (see also Appendix B) leads to the equation

$$\rho(t) = \rho(t=0) + \int_0^t d\tau \, F[\rho(\tau)] + \int_0^t dW_\tau \, G[\rho(\tau)], \tag{2.7}$$

where W_τ is a Wiener process. The integral $\int_0^t dW_\tau \; G(\rho(\tau))$ can be integral of Itô kind or integral of Stratonovich kind (for more discussion see Appendix B). In the next two sections we shall discuss these two cases.

We note that the choice between Itô and Stratonovich interpretation of the stochastic integral in Eq. (2.7) depends on the characteristics of noise in the studied system. Thus the choice of the interpretation has to be checked by comparing

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