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Chaotic quantum behaved particle swarm optimization algorithm for solving nonlinear system of equations

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ABSTRACT

This study proposes a novel chaotic quantum behaved particle swarm optimization algorithm for solving nonlinear system of equations. Different chaotic maps are introduced to enhance the effectiveness and robustness of the algorithm. Several benchmark studies are carried out. Logistic map gives the best results and is utilized in solving nonlinear equation sets. Nine well known problems are solved with our algorithm and results are compared with Quantum Behaved Particle Swarm Optimization, Intelligent Tuned Harmony Search, Gravitational Search Algorithm and literature studies. Comparison results reveal that the proposed algorithm can cope with the highly non-linear problems and outperforms many algorithms which exist in the literature.

1. Introduction

Solving nonlinear sets of equations is a hard and tedious task as these kind of problems appear in many real-world applications. Economics [1], chemistry [2], physics [3–6] are some of the examples of scientific areas applied to solve these type of equations. Newton-type methods which are a conventional procedure for solving system of nonlinear equations are derivative based and depend on the sensitivity of the initial value. These are the main drawbacks of Newton's method since some of the derivatives do not come into existence and wrong initial guesses will lead to unexpected results. Bader [7] claimed that the Newton–Raphson method is incapable of solving a large scale system due to its high memory requirements. Bader [7] also proposed a tensor method utilizing Krylov subspace methods for solving nonlinear sets. Two different solution strategies called interval and continuation methods have been proposed for this task. Interval methods [8–18] are robust and tend to slow [19]. Continuation methods [20,21] are more effective for problems which has lower total degrees [19]. Plenty of research activities devoted to a successful solution on this problem have been made, however there is still room to improve existing studies.

New approaches on this issue arise from metaheuristic applications. Karr et al. [22] hybridized the Genetic algorithm and Newton's method for solving a nonlinear system of equations. The local search procedure is maintained by the Genetic algorithm which supplies initial values of Newton's method. Wang et al. [23] proposed a modified particle swarm equation with controller to improve the search dynamics of traditional PSO. Ouyang et al. [24] hybridized the Nelder–Mead algorithm with Particle Swarm Optimization to solve systems of nonlinear equations. Selection of good initial values of the Nelder–Mead

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algorithm is supplied by the particle swarm method which suffers from being trapped by local minima. Jia and He [25] combined Artificial Bee Colony with Particle Swarm Optimization to improve the effectiveness of the search mechanism. Yang et al. [26] hybridized the Hooke–Jeeves algorithm with the Glow-worm Swarm Optimization algorithm to speed up the local search procedure. Results showed that the modified algorithm has high convergence rate and accuracy for solving nonlinear equations. Hirsch et al. [27] proposed the Continuous Greedy Randomized Adaptive Search Procedure (C-GRASP) for solving nonlinear set of equations. Mo et al. [28] postulated Conjugate Direction Particle Swarm Optimization (CDPSO) which introduces the conjugate direction method into particle swarm optimization in order to enhance the ability of solving high dimensional optimization problems. Toutounian et al. [29] proposed a hybrid scheme which combines the Electromagnetic Metaheuristic method with the finite difference version of the Newton–GMRES method. Sacco and Henderson [30] introduced a novel search strategy that combines Luus–Jaakola random search with Fuzzy Clustering Means. Promising solution areas are obtained by this procedure and the Nelder–Mead algorithm is applied on these areas to reach an optimum solution. Moreover, the Genetic algorithm [31–35], Evolutionary algorithm [36,37], Firefly algorithm [38], Artificial Bee Colony algorithm [39], Invasive weed optimization [40], Imperialist competitive algorithm [41] and Particle Swarm Optimization [42,43] are also utilized for sorting out nonlinear systems of equations.

There is an increased interest on advances in applications of nonlinear dynamics, generally, on the use of chaos in optimization algorithms. Randomly generated chaotic sequences have been incorporated with majority of the metaheuristics to upgrade the probing potential of the mentioned algorithms such as Bee Colony algorithm [44], Bat algorithm [45], Harmony search [46], Krill Herd algorithm [47], Firefly algorithm [48], Imperialist competitive algorithm [49], Genetic algorithms [50], Simulated annealing [51], Ant colony optimization [52,53], Big Bang–Big Crunch algorithm [54] and Particle swarm optimization [55–60]. In order to solve this high dimensional optimization problem, we propose the Chaotic Quantum behaved Particle Swarm Optimization method. We used different chaotic maps to replace random parameters which exist in QPSO. In this way, different pseudorandom number sequences have been generated and search capacity of the algorithm has increased. To test the effectiveness of the proposed algorithms, we applied some benchmark functions including Colville, Schaffer, Griewank, Rastrigin, Dropwave and Rosenbrock functions. Chaotic map which gives the best results will be selected and utilized in solving nonlinear system of equations.

System of non linear equations can be described as

$$\begin{aligned} f_1(x_1, x_2, x_3, \ldots, x_n) &= 0.0 \\ f_2(x_1, x_2, x_3, \ldots, x_n) &= 0.0 \\ &\vdots \\ f_m(x_1, x_2, x_3, \ldots, x_n) &= 0.0, \end{aligned} \tag{1}$$

where $f_i, i = 1, 2, \ldots, m$ is a nonlinear equation system and $X = (x_1, x_2, \ldots, x_n)$ is the unknown solution vector. The problem is transformed into optimization given as

$$\text{Minimize } F(X) = \sqrt{\sum_{i=1}^{m} f_i^2}. \tag{2}$$

The paper is organized as follows. Section 2 describes Particle Swarm and Quantum behaved Particle Swarm Optimization methods, Section 3 introduces the chaotic maps which exist in the literature, Section 4 gives the CQPSO method proposed for solving the system of equations, in Section 5 numerical tests are studied and Section 6 gives the conclusion of this research.

2. Particle swarm optimization

2.1. Basics of particle swarm optimization

Particle Swarm Optimization [61] is a population based algorithm constructed on swarm behaviour of fish schooling and bird flocking to find an optimum solution of a problem. The algorithm has some similarities with Genetic algorithms and ant algorithms; however it is much simpler since it does not need crossover or mutation operator [62]. Each candidate solution named as "particle" flies around the solution space and lands on the optimal position. Particles in the swarm adjust their position by their own experience and experience of neighbouring particles (agents). Each agent has a memory which keeps track of its previous best position P_{best} with its respective fitness value. Agent with the best fitness value in the swarm is called Global best G_{best} which holds the optimum solution for this generation. Assume a swarm with N dimensional population with D dimensional particles. Position and velocity vectors of the ith particle is represented with $x_i = (x_{i1}, x_{i2}, \ldots, x_{iD})$ and $v_i = (v_{i1}, v_{i2}, \ldots, v_{iD})$ where $i = 1, 2, 3, \ldots, N$. Position and velocity update are maintained by

$$v_i^{k+1} = w^k v_i^k + c_i r_1 (P_{bi}^k - x_i^k) + c_2 r_2 (P_g^k - x_i^k) \tag{3}$$

$$x_i^{k+1} = x_i^k + v_i^{k+1}, \tag{4}$$