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An improved momentum exchanged-based immersed boundary–lattice Boltzmann method by using an iterative technique

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ABSTRACT

A novel immersed boundary–lattice Boltzmann method (IB–LBM) is proposed for incompressible viscous flows in complex geometries. Based on the momentum exchanged-based IB–LBM, an iterative technique is introduced to enforce the non-slip boundary condition at the boundary points. Moreover, the proposed IB–LBM overcomes the drawback that the numerical results of the previous work (Wu and Shu, 2009) which is affected by the distribution of Lagrangian points. A simple theoretical analysis is developed to obtain the optimal iteration parameters. Numerical results show that the present scheme has second-order accuracy and is not affected by the distribution of Lagrangian points. It also shows that the non-slip boundary condition is satisfied on the boundary. This verifies that our IB–LBM is capable of simulating flow problems with complex boundaries.

1. Introduction

The efficiency of numerical methods depends on the quality of the mesh. Compared with the complex unstructured grids, generating a Cartesian grid is very simple. Cartesian grid methods which are applied widely in numerical simulation are easy to be implemented as well as adopting fast algorithms. However, Cartesian grid techniques are difficult to effectively simulate flow problems in complex geometries. In recent years, the immersed boundary method (IBM) has received a great attention for its simplicity. IBM was first introduced by Peskin [1] to solve the problem of blood flow in the heart. This method applies a set of Lagrangian points to represent the complex boundaries. And the governing equations of flow field can be discretized and solved on a Cartesian Eulerian grid. The interaction between Eulerian points and Lagrangian points is calculated in terms of distribution and interpolation operations by using the smoothed Dirac delta function. When boundaries move, this scheme only needs to track the positions of Lagrangian points. Thus IBM simplifies the treatment of complex boundaries and improves the computational efficiency greatly.

The key issue of IBM is the calculation of force density at the boundary points. IBM now has been developed into a major category of algorithms. Several recent influential works about it are introduced in the following. Goldstein et al. [2] proposed a feedback-forcing formulation which needs to adjust two free parameters and also requires a small time step.

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Lai and Peskin [3] developed a formal second-order accuracy IBM by applying the penalty method and high-order Dirac delta function. However, it does not achieve truly second-order accuracy. Fadlun et al. [4] proposed a direct-forcing scheme which is used widely because it does not have any free parameters and allows a large time step. Kim et al. [5] developed an IBM by introducing a mass source/sink as well as momentum forcing which is based on the finite-volume approach. Roma et al. [6] studied an adaptive version of IBM in order to enhance the accuracy of IBM by using a coupled multilevel adaptive technique. Griffith et al. [7] proposed another adaptive version of IBM to enhance boundary layer resolution.

IBM is a solver for the boundary condition, which can be combined with any fluid solvers. As an alternative computational method for the conventional N–S (Navier–Stokes) solver, the lattice Boltzmann method (LBM) is based on mesoscopic which can be derived from LGA (Lattice gas automata) [8]. And it can also be viewed as a finite difference scheme of the continuous Boltzmann equation [9]. Compared with the N–S solver, LBM is a particle-based method, so it has intrinsic superiority for parallel computing. Moreover, it need not to solve the Poisson equation for the pressure field. Furthermore, the program implementation of LBM is relatively easy. Thus, this method is popular in recent years.

The combination of IBM and LBM (IB–LBM) has received wide attention for its efficiency. Feng et al. [10] had done a pioneering work for the coupling algorithm. They successfully simulated some fluid–structure interaction problems. In the early work, they calculated the interaction force between fluid and particles by the penalty method which introduces a user-defined spring parameter. Then they proposed the direct method without free parameters [11]. Soon afterwards Niu et al. [12] proposed a momentum exchanged-based IB–LBM to simulate some incompressible viscous flow problems. The version of this IBM is very simple and the force density is calculated by using the momentum exchange rules. The ideas of the direct-forcing and the momentum-exchange are simple and physically plausible. However, as explicit schemes, the non-slip boundary condition is often unable to be satisfied on the boundary. The direct consequence is that streamlines penetrate the immersed boundary. Recently, Wu et al. [13] proposed a boundary condition-enforced IBM. Their basic idea is that the corrected velocity field satisfies the non-slip boundary condition. This method is an implicit scheme. Wu et al. apply this method to simulate a variety of flow and heat transfer problems [14–16]. However, the coefficient matrix has been introduced in this scheme. As a result, excessive Lagrangian points would increase the condition number and singularity of the coefficient matrix. And the linear algebraic equation would be difficult to solve. This is a drawback of this method.

The purpose of this paper is to construct a novel robust IB–LBM. We propose the version of IB–LBM which is based on the following principles:

- The non-slip boundary condition must be satisfied on the boundary.
- The performance of IBM is not affected by the distribution of Lagrangian points.

The foundation of the present method is the momentum exchanged-based IBM [12]. In order to enforce the non-slip boundary condition on the boundary, an iterative method is introduced. The iterative technique is first proposed by Luo et al. [17,18] in their IBM. In our iterative method, the incremental of force density is calculated by using the momentum exchanged rules. By choosing an appropriate relaxation parameter, we can prove that the iteration converges. Because the present IB–LBM avoids the procedure of matrix inversion, the performance of this method is not affected by the distribution of Lagrangian points. Thus, the present method is robust.

2. Numerical method

2.1. Review of the immersed boundary method and lattice Boltzmann method

As shown in Fig. 1, a two-dimensional domain Ω for the viscous incompressible flows is considered. It contains a closed immersed boundary, which divides the flow domain into internal and external areas. In accordance with the previous approaches, internal and external areas are the same as the computational domain. The effect of immersed boundary is loaded as a force term in the momentum equation. The macroscopic governing equations can be written as

$$\nabla \cdot \mathbf{u} = 0, \tag{1}$$

$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} \right) = -\nabla p + \mu \nabla^2 \mathbf{u} + \mathbf{f}, \tag{2}$$

with the boundary condition on Γ

$$\mathbf{u}(\mathbf{X}(s,t)) = \mathbf{U}_B(\mathbf{X}(s,t)), \tag{3}$$

where $\rho, \mathbf{u}, p$ represent density, velocity and pressure, respectively, and $\mathbf{U}_B$ is the velocity on the immersed boundary. Physical parameter μ is the dynamics viscosity. $\mathbf{f}$ denotes the force term which comes from the immersed boundary, and it can be expressed as

$$\mathbf{f}(\mathbf{x},t) = \int_\Gamma \mathbf{F}(\mathbf{X}(s,t))\delta(\mathbf{x} - \mathbf{X}(s,t))ds, \tag{4}$$

where $\mathbf{x}$ is the coordinate of Eulerian node, and $\mathbf{X}$ is the coordinate of Lagrangian node. $\mathbf{F}(\mathbf{X}(s,t))$ is the force density on the immersed boundary, and $\delta(\mathbf{x} - \mathbf{X}(s,t))$ denotes the Dirac delta function.