



Dynamics of a general prey–predator model with prey-stage structure and diffusive effects



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ABSTRACT

In this paper, we propose and study the dynamics of a diffusive prey–predator model with general functional response and stage-structure for the prey. Firstly, we consider the asymptotical stability of equilibrium points and Hopf bifurcation for the reduced ODE system. Secondly, the existence and uniform boundedness of global solutions and stability of equilibrium points for the corresponding reaction–diffusion system are discussed. Finally, we establish the existence and the nonexistence of nonconstant positive steady states of this reaction–diffusion system, which indicates the effect of large diffusivity. Our results shows the importance of the diffusion rate of the predator species (i.e., d_3). The large diffusion rate of the predator alone will help the generation of patterns. However, a large diffusion rate of the immature prey species or a large diffusion rate of the mature prey species can lead to the nonexistence of spatial patterns.

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1. Introduction and the mathematical model

Many kinds of predator–prey models have been studied extensively (see, [1–4]). In the natural world, there are many species whose individual members have a life history that takes them through two stages: immature and mature. In particular, we have in mind mammalian populations and some amphibious animals, which exhibit these two stages. Population growth models that include age stage or body size structure often predict complex population dynamics. Due to the above realistic evidences, the stage-structured models have received much attention in recent years, see, [4–22] and the references therein. Generally speaking, population growth models that include stage structure predict more complex population dynamics than those without taking these factors into account.

In [5], the authors have proposed and investigated the following of a Lotka–Volterra predator–prey model with prey-stage structure

$$\begin{aligned}\frac{dx_1}{dt} &= Bx_2 - Cx_1 - D_1x_1 - \gamma x_1^2 - kx_1y, \quad t > 0, \\ \frac{dx_2}{dt} &= Cx_1 - D_2x_2, \quad t > 0, \\ \frac{dy}{dt} &= y(-D_3 + \delta_1 kx_1 - \eta y), \quad t > 0,\end{aligned}$$

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where x_1, x_2 are the population densities of immature and mature prey species, respectively. y denotes the density of predator population. $B, C, D_1, D_2, D_3, \gamma, \delta_1$ are positive constants. B represents the birth rate of the immature prey, C denotes the transmission rate from immature prey individuals, γ is the intra-specific competition rate of the immature prey, D_1 and D_2 represent the death rates of immature and mature prey, respectively. D_3 is the death rate of predator, δ_1 is the conversion rate. *Predators may be generalists that consume prey species other than the one in the model, or specialists that depend exclusively on the prey species in the model. Generalist predators can survive even when the prey species is not present. In models for generalist predators that means the linear growth term should be positive (namely, $\eta > 0$). For specialist predators, the predator cannot survive without the prey, so the predator population will go to zero if the prey is not present. At this moment, $\eta = 0$. Hence, η is nonnegative constant.*

On the other hand, in order to understand the dynamics of a predator–prey model involves not only the size and structure of the population, but also the ability to capture prey and renew itself. One significant component of the predator–prey relationship is the predator functional response, i.e., the rate of prey consumption by an average predator. Generally, the functional response can be classified into two types: *prey-dependent* and *predator-dependent*. Prey-dependent indicates that the functional response is only a function of the prey density, while predator-dependent means that the functional response is a function of both the prey and the predator densities. The classical Holling types I–III, the Holling type IV (or Monod–Haldane type), the Ivlev type and Rosenzweig type are strictly prey-dependent functional responses; ratio-dependent type, Hassell–Varley type, Beddington–DeAngelis type as well as Crowley–Martin type are predator-dependent functional responses. We note that an important factor in modeling of predator–prey is the choice of functional responses governing the prey–predator interactions.

Motivated by the papers in [5], in this paper we propose a diffusive prey–predator model with stage structure for the prey. Let X and y be densities of prey and predator, respectively. A Lotka–Volterra prey–predator model with logistic growth in the prey and Malthusian decay and with in the predator general functional response is

$$\begin{aligned}\frac{dX}{dt} &= EX - FX^2 - \phi(X)y, \quad t > 0, \\ \frac{dy}{dt} &= y(-D_3 + \delta_1\phi(X)), \quad t > 0.\end{aligned}\tag{A}$$

Now, we divide prey X into two parts: the immature x_1 and mature x_2 , and so $X = x_1 + x_2$. In the real world, predator consumes immature prey in general. For some special cases, the possibility being consumed for the immature is larger than that for the mature. Taking into account this fact and in view for simplicity, we make the following assumptions:

- (A₁): The birth rate of the immature population is proportional to the existing mature population with a proportionality constant B ; for the immature population, the death rate and transformation rate of mature are proportional to the existing immature population with proportionality constants C and D_1 ; the immature population is density restriction ($\gamma > 0$);
- (A₂): The immature prey do not yield offsprings;
- (A₃): The predator only preys on immature prey, and with generalize functional response function, which depends on the numbers of immature prey; the predators are specialists that depend exclusively on the prey species in the model.

Under the above assumptions, we establish the ODE prey–predator model with general functional response and stage-structure for the prey as follows

$$\begin{aligned}\frac{dx_1}{dt} &= Bx_2 - Cx_1 - D_1x_1 - \gamma x_1^2 - \phi(x_1)y, \quad t > 0, \\ \frac{dx_2}{dt} &= Cx_1 - D_2x_2, \quad t > 0, \\ \frac{dy}{dt} &= y(-D_3 + \delta_1\phi(x_1)), \quad t > 0.\end{aligned}\tag{1.1}$$

Furthermore, we assume that the functional response $\phi(x)$ satisfies:

- (H)′: $\phi(0) = 0$, $\phi'(x) > 0$ ($x \geq 0$), and $0 < \phi(x) < L$, where L is a positive constant.

The second condition in (H)′ implies that, as the prey population increases, the consumption rate of prey per predator increases. Some explicit forms for the predator functional response that have been used are

$$\begin{aligned}\phi(x) &= L_1(1 - e^{-px}) \quad [\text{Ivlev (1961)}]; \\ \phi(x) &= \frac{L_1x}{a + x} \quad [\text{Holling type II (1965)}]; \\ \phi(x) &= \frac{L_1x^2}{a + x^2} \quad [\text{Holling type III (1965)}]; \\ \phi(x) &= L_1x^q (q < 1) \quad [\text{Rosenzweig (1971)}].\end{aligned}$$

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