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Explicit two-step peer methods

Rüdiger Weiner[a,*], Katja Biermann[b], Bernhard A. Schmitt[c], Helmut Podhaisky[a]

[a] Fachbereich Mathematik und Informatik, Universität Halle, D-06099 Halle, Germany
[b] Institut für Mathematik, TU Berlin, D-10623 Berlin, Germany
[c] Fachbereich Mathematik und Informatik, Universität Marburg, D-35032 Marburg, Germany

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Abstract

We present a new class of explicit two-step peer methods for the solution of nonstiff differential systems. A construction principle for methods of order $p = s$, s the number of stages, with optimal zero-stability is given. Two methods of order $p = 6$, found by numerical search, are tested in Matlab on several representative nonstiff problems. The comparison with ODE45 confirms the high potential of the new class of methods.

Keywords: Explicit peer methods; Nonstiff ODE systems; Zero stability; Stability region; General linear methods

1. Introduction

Many efficient methods have been developed for the numerical solution of ordinary differential equations

$$y' = f(t, y), \qquad y(t_0) = y_0 \in \mathbb{R}^n, \quad t \in [t_0, t_e]. \tag{1}$$

The most important classes of methods are one-step and multistep methods and efficient computer codes from each of these classes are widely available. To these belong the one-step code DOPRI5(4) [1], which is also the basis of the MATLAB routine ODE45 [2], and the multistep code VODE [3]. The respective advantages of one-step and multistep methods are well known, cf. the text book of Hairer, Nørsett and Wanner [4]. With parallel implementation in mind the new class of peer methods has been introduced by some of the authors in a series of papers, e.g. [5–7], which concentrated on stiff problems and linearly-implicit and implicit two-step methods. The new feature of peer methods is that they possess several stages like Runge–Kutta-type methods, but all of these stages have the same properties and no extraordinary solution variable is used. These methods combine the positive features of both the Runge–Kutta and multistep methods, having good stability properties and no order reduction for very stiff systems. In numerical tests on parallel computers they were rather efficient, e.g. [8,7], and also in sequential computing environments they were competitive with standard codes [9].

* Corresponding author.
E-mail addresses: weiner@mathematik.uni-halle.de (R. Weiner), biermann@math.TU-Berlin.de (K. Biermann), schmitt@mathematik.uni-marburg.de (B.A. Schmitt), podhaisky@mathematik.uni-halle.de (H. Podhaisky).

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In this paper we now discuss explicit two-step peer methods for nonstiff problems and concentrate on non-parallel methods. "Classical" explicit two-step Runge–Kutta schemes have been considered for instance by Jackiewicz and Zennaro [10] and Tracogna and Welfert [11], and in the context of general linear methods by Jackiewicz and Tracogna [12] and Wright [13]. Numerical results of certain parallel explicit two-step methods are given in [14].

The paper is organized as follows: In Section 2 we formulate the new class of explicit two-step peer methods as a special case of linearly-implicit peer methods. In this paper we discuss a general class containing parallel and sequential methods.

In Section 3 we give order results using simplifying conditions and consider stability properties. By a special choice of coefficients we ensure optimal zero-stability for arbitrary stepsize sequences and avoid some of the theoretical difficulties of linearly implicit peer methods; see [7]. We describe a strategy for the construction of zero stable methods of order $p = s$.

Section 4 is devoted to the numerical search for good methods using the remaining free coefficients. We present two methods of order $p = 6$ with six stages and display their stability regions. Furthermore, we discuss the implementation of the methods with stepsize control.

Numerical results of a MATLAB code for several widely accepted nonstiff test problems are reported in Section 5. Comparisons with the MATLAB code ODE45 show the efficiency of the proposed methods.

Finally we give some conclusions and an outlook for future work.

2. Derivation of the methods

In [7] linearly-implicit two-step peer methods were considered. In each time step from t_m to $t_{m+1} = t_m + h_m$ solutions $Y_{m,i} \cong y(t_{m,i})$, $i = 1, \dots, s$, are computed as approximations at the points

$$t_{m,i} := t_m + h_m c_i, \quad i = 1, \dots, s. \tag{2}$$

In these methods for stiff systems an approximation T to the Jacobian is used. By setting $T = 0$ we obtain an explicit method, which can be applied to nonstiff systems.

In contrast to [7] in this paper we will not focus on parallel methods. Therefore, as in explicit Runge–Kutta schemes, it is natural to use also the previously computed stage values from the present step:

$$Y_{m,i} = \sum_{j=1}^{s} b_{ij} Y_{m-1,j} + h_m \sum_{j=1}^{s} a_{ij} f(t_{m-1,j}, Y_{m-1,j}) + \sum_{j=1}^{i-1} q_{ij} Y_{m,j} + h_m \sum_{j=1}^{i-1} r_{ij} f(t_{m,j}, Y_{m,j}), \quad i = 1, \dots, s.$$

With the notations

$$Y_m = (Y_{m,i})_{i=1}^{s} \in \mathbb{R}^{sn}, \qquad F(Y_m) = (f(Y_{m,i}))_{i=1}^{s}, \qquad A = (a_{ij}), \qquad B = (b_{ij}),$$
$$Q = (q_{ij}), \qquad R = (r_{ij})$$

the methods can be written in the compact form (for simplicity for autonomous equations)

$$Y_m = (B \otimes I) Y_{m-1} + h_m (A \otimes I) F(Y_{m-1}) + (Q \otimes I) Y_m + h_m (R \otimes I) F(Y_m),$$

where Q and R are strictly lower triangular matrices.

In the following, for ease of representation only, we consider scalar equations. By replacing the coefficient matrices according to

$$B := (I - Q)^{-1} B, \quad A, R \text{ analogously}$$

these methods simplify to

$$Y_m = B Y_{m-1} + h_m A F(Y_{m-1}) + h_m R F(Y_m). \tag{3}$$

Remark 1. Sequential linearly-implicit two-step methods were considered in [9]; however, (3) differs from that class when $T = 0$ is used there. □