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Modification of the Engelund bed-load formula

Zhen Meng, Danxun Li*, Xingkui Wang

State Key Laboratory of Hydroscience and Engineering, Tsinghua University, Beijing 100084, China

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ABSTRACT

The classic Engelund bed-load formula involves four oversimplified assumptions concerning the quantity of particles per unit bed area that can be potentially entrained into motion, the probability of sediment being entrained into motion at a given instant, the mean velocity of bed-load motion, and the dimensionless incipient shear stress. These four aspects are reexamined in the light of new findings in hydrodynamics, and a modified bed-load formula is then proposed. The modified formula shows promise as being reliable in predicting bed-load transport rates in a wide range of flow intensities.

1. Introduction

Bed-load transport occurs as the motion of sediment particles along the channel bed by rolling, sliding, and/or saltating (Chien & Wan, 1983). As it plays an important role in a variety of scientific and engineering settings, bed-load transport has remained a key research interest in the hydraulic community for more than a century.

To predict bed-load transport rate, numerous formulas have been proposed by researchers from across the world. Existing bed-load formulas can be generally classified into two categories, i.e., empirical and semi-empirical (Zanke, 2001). Empirical formulas, such as those by Cheng (2002), Meyer-Peter and Müller (1948), Parker (1978), and Knack and Shen (2015), are based purely on regressive analysis of measured data. Semi-empirical formulas, in contrast, involve both theoretical derivations and statistical analysis. Typical semi-empirical formulas include those proposed by Ashida and Michiue (1972), Bagnold (1973), Bialik and Czernuszenko (2013), Einstein (1950), Engelund and Fredsøe (1976), Fredsøe and Deigaard (1992), Lajeunesse et al. (2010), Soulsby and Damgaard (2005), Van Rijn (1984), Wilson (1987), Yalin (1977), and Zhong et al. (2012).

Popular bed-load formulas, either empirical or semi-empirical, perform generally well within their domains of validity. Outside the domain of validity, however, even a well-established formula may over-predict or under-predict real bed-load transport by several orders of magnitude (Recking, 2010; Talukdar et al., 2012; Yu et al., 2009, 2012). For example, the Einstein (1950) formula fits well with measured data at weak to moderate transport rate, but deviates considerably at high transport rate (Wang et al., 2008). The applicability of classical bed-load formulas can be extended by modifying some assumptions and oversimplifications made in the original derivations in the light of new findings in hydrodynamics. Examples of such extensions include the modification of the Meyer-Peter and Müller (1948) formula by Huang (2010) and Wong and Parker (2006), and the modification of the Einstein (1950) formula by Armanini et al. (2014), Sun and Donahue (2000), Wang et al. (2008), and Yalin (1977).

In the present study, we try to modify the Engelund formula. The classical formula starts from the universal relationship for sediment flux in volume, q_b:

$$q_b = \frac{\pi}{6} D^3 N_b P u_b \tag{1}$$

where D is particle diameter, N_b is the quantity of particles per unit bed area that can be potentially entrained to motion, P is probability of sediment particle to be entrained into motion at any instant, and u_b is mean velocity of moving particles in flow direction.

The oversimplification of N_b, u_b, P, and Θ_c (Θ_c is dimensionless incipient shear stress) leads to drawbacks in the Engelund formula. The drawbacks have been pointed out by several researchers, such as Chien and Wan (1983), Fredsøe and Deigaard (1992), Qu (1998), and Zhang and Mcconnachie (1994), but none of these researchers offered satisfactory modification to the original formula. The present

* Correspondence to: State Key Laboratory of Hydroscience and Engineering, Department of Hydraulic Engineering, Tsinghua University, Beijing 100084, China. Tel.: +86 10 62781747.

E-mail addresses: edison9981@gmail.com (Z. Meng), lidx@mail.tsinghua.edu.cn, lidx@tsinghua.edu.cn (D. Li), wangxk@mail.tsinghua.edu.cn (X. Wang).

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Notation

D	diameter of particle (L)
g	gravitational acceleration (L T^{-2})
q_b	bed-load transport rate in volume per unit width (L^2 T^{-1})
Φ	dimensionless transport rate
K	modified coefficient of Φ
u_*	friction velocity corresponding to skin friction (L T $^{-1}$)
u_{*c}	incipient friction velocity (L T^{-1})
u_f	flow velocity near bed (L T $^{-1}$)
u_b	mean velocity of bed-load (L T $^{-1}$)
x	constant coefficient
y	constant coefficient
z	constant coefficient
λ	constant coefficient
α	constant coefficient
m	constant coefficient
k	constant coefficient
μ	a varied number
σ	a coefficient
κ	constant coefficient
P	sediment entrainment probability
A_*	constant coefficient
B_*	constant coefficient
η_0	constant coefficient
β	dynamic friction coefficient of submerged sediment particles
ρ	density of fluid (M L^{-3})
ρ_s	density of sediment (M L^{-3})
Θ	dimensionless shear stress
Θ_c	dimensionless incipient shear stress
δ_b	thickness of bed-load layer (L)
N_b	quantity of particles per unit bed area that can be potentially entrained into motion
n_b	quantity of moving sediment particles per unit bed area
C_D	drag coefficient
C_L	lift coefficient
F_D	fluid drag force (M L T^{-2})
F_L	fluid lift force (M L T^{-2})
F_G	submerged weight (M L T^{-2})
w	particle fall velocity of bed material (L T^{-1})
k_s	equivalent roughness of Nikuradse (L)
I_d	index of agreement
$RMSRE$	root mean square relative error
R	discrepancy ratio

paper will modify some assumptions and propose a new version of the classical Engelund bed-load formula.

2. Modification of the Engelund formula

2.1. Determination of N_b

Engelund assumed that the quantity of particles per unit bed area that can be potentially entrained to motion is as follows:

$$N_b = \frac{1}{D^2} \tag{2}$$

Such an assumption, while reasonable at low transport intensity, is improper in flows with high shear stress (Zhang & Mcconnachie, 1994). At low shear stress, bed-load particles move roughly in a single layer; when the flow becomes sufficiently powerful, however, bed-load transport becomes multi-layered, or even in sheet, with a thickness of δ_b larger than particle diameter (Abrahams, 2003; Nnadi & Wilson, 1992; Sumer et al., 1996; Seizilles et al., 2014; Van Rijn, 1984; Wilson, 1987, 1989). To correct the original N_b, the following relationship is recommended:

$$N_b = \frac{\delta_b}{D}\frac{1}{D^2} \tag{3}$$

Considerable progress has been made in quantifying δ_b. For example, Einstein (1950) assumes $\delta_b = 2D$ regardless of flow conditions, Van Rijn (1984) equals δ_b to the saltation height with a maximum of $10D$, and Wilson (1987) proposes $\delta_b = 10\Theta D$, where $\Theta = \rho u_*^2/(\rho_s - \rho)gD$ is dimensionless shear stress (ρ is the fluid density, ρ_s is the sediment density, u^* is the friction velocity corresponding to skin friction, and g is the gravitational acceleration). Here we introduce a new assumption for "effective" thickness of bed-load layer as follows:

$$\delta_b/D = m(\Theta - \Theta_c) \tag{4}$$

where m is a constant coefficient.

Substituting Eq. (4) into Eq. (3) yields a new version of N_b:

$$N_b = m(\Theta - \Theta_c)\frac{1}{D^2} \tag{5}$$

2.2. Determination of P

In the original Engelund formula, the probability P is determined as follows:

$$P = \frac{6}{\pi\beta}(\Theta - \Theta_c) \tag{6}$$

where β is the dynamic friction coefficient of submerged sediment particles. The fact that calibration against measured data yields P greater than unity in flows of $\Theta > 0.5$ prompted Engelund to revise Eq. (6) to the following form:

$$P = \left[1 + \left(\frac{\pi\beta/6}{\Theta - \Theta_c}\right)^4\right]^{-0.25} \tag{7}$$

Unfortunately, introduction of Eq. (7) leads to a disastrous under-prediction of high transport rates (Chien & Wan, 1983; Zhang & Mcconnachie, 1994).

Here we believe that the approach proposed by Einstein (1950) to determine P is both theoretically sound and practically feasible. Einstein (1950) assumed that P corresponds to the possibility of "the dynamic lift on the particle is larger than its submerged weight", which can be determined as follows:

$$P = 1 - \frac{1}{\sqrt{\pi}}\int_{-B_*/\Theta - 1/\eta_0}^{B_*/\Theta - 1/\eta_0} e^{-t^2}dt \tag{8}$$

where $B_* = 1/7.0$ and $\eta_0 = 1/2$ are constant coefficients.

With Eqs. (5) and (8) we can calculate the number, n_b, of particles in motion per unity bed area at any given instant such that

$$n_b = N_b P = mP(\Theta - \Theta_c)/D^2 \tag{9}$$

Lajeunesse et al. (2010) recently reported experimental results, showing that n_b increases linearly with $(\Theta - \Theta_c)$ as follows:

$$n_b = \mu(\Theta - \Theta_c)/D^2 \tag{10}$$