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Theory and application of nonlinear river dynamics

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Abstract

A theoretical model for river evolution including riverbed formation and meandering pattern formation is presented in this paper. Based on nonlinear mathematic theory, the nonlinear river dynamic theory is set up for river dynamic process. Its core content includes the stability and tropism characteristics of flow motion in river and river selves' evolution. The stability of river dynamic process depends on the response of river selves to the external disturbance, if the disturbance and the resulting response will eventually attenuate, and the river dynamics process can be restored to new equilibrium state, the river dynamic process is known as stable; otherwise, the river dynamic process is unstable. The river dynamic process tropism refers to that the evolution tendency of river morphology after the disturbance. As an application of this theory, the dynamical stability of the constant curvature river bend is calculated for its coherent vortex disturbance and response. In addition, this paper discusses the nonlinear evolution of the river peristaltic process under a large-scale disturbance, showing the nonlinear tendency of river dynamic processes, such as river filtering and butterfly effect.

Key Words: River, Dynamic process, Reservoir, Nonlinear, Long term evolution, River dynamics

1 Introduction

With the advance of computer technology, computational models are often used in engineering practices to forecast the long-term evolution process of natural rivers and their responses to a disturbance (e.g., engineering structure). The nonlinear analysis has been applied to investigate the engineering stability problems and to explore the bifurcation behavior and the chaotic phenomenon of engineering system after the alteration of some parameters (Hu, 1992; Hao, 1992; Chen, 1993; Hu and Guo, 2004; Zhou and Zhao, 2004; and Gao, 2005). Recently, when studying river dynamic process, associated with serious ecological problem of rivers in recent years, a series of river philosophical issues are raised (Wang et al., 2013). For instance, what are the principles that govern the responses of river to man-made disturbance? What is the most stable river morphology? What is the most stable bedform (Xu, 1985 and 1989; Imran et al., 1999; Wang and Cao, 2001; Ni and Wang, 2000; Shao and Wang, 2002; Bai, 2006 and 2007; Zhang and Hu, 2007; Wang, 2006; and Wang et al., 2007) and what is the most stable river pattern (B.C. Eaton Robert, et al., 2010; Lucy Clarke et al., 2010)?

River dynamic process is affected by many factors. First, it is influenced by the river basin, which can be considered as the external factors. Being an integral part of the basin, the geometric and dynamic properties of a river are determined by basin characteristics. The combined effects of various factors within the basin, such as climate, topography, lithology, soil properties, and vegetation cover and land use, determine the runoff and sediment yield and consequently river dynamic process. Second, it is affected by its own physical characteristics, namely the internal factors. As the water flows through, the sediment erosion, transport and deposition processes constitute the main body of the river dynamic process. Sediment movement under different flow structure leads to different evolution processes of the river bed and bank forms (Duan J. G., Wang S. S. Y, Jia Y, 2001; Bai and Xu, 2005; Duan J. G., 2005, Bai et al., 2006; Xu, 2006; Jang C. L., Shimizu Y., 2007, Bai et al., 2008; Xu, 2008; Xu and Bai, 2008, Alessandra Crosat, 2008; Xu and Bai, 2010; Bai and Xu, 2010; Xu et al., 2010).

River dynamic process refers to the formation and evolution of river under natural and man-made conditions. As part of the basin, river characteristics are influenced by those of basin, resulting in variations in the river dynamic processes, leading to different river morphology in the world. However, this dynamic evolution is not haphazard, but has certain rules, such as the automatic adjustment of alluvial rivers. With the ability of this automatic adjustment, the water and

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sediments from the upstream of river are discharged to downstream, associated with the energy consumption of the river, so that the energy in river remained relatively balanced, resulting in a certain degree of stability of the river. The coexistence of randomness and certainty in rivers is the nonlinear characteristic of river dynamics (Bai et al., 2006; Wang et al., 2006; and Wang et al., 2007).

The river dynamic process can be divided into four levels, in accordance with gradually various scale sizes: (1) bedform evolution (sand ripples, sand dunes) and river regime unit evolution (Staggered bars, point bars, deep grooves, shallow beaches and local deltas, etc.); (2) rivers local and whole plane evolution (i.e. river bend changing, channel meandering or branching ect.). These different levels of the river dynamic process can be expressed by weak river dynamic process, strong river dynamic process and transitional river dynamic process respectively. The river dynamic process is a typical nonlinear process, with a series of significant nonlinear characteristics (Han Q. 2008; Bai and Yang, 2011; Bai and Xu, 2012; Xu and Bai, 2012; B. C. Eaton Robert, et al, 2010; Lucy Clarke et al., 2010).

Attempting to answer what are the principles that govern the responses of river to man-made disturbance? What is the most stable river morphology? What is the most stable bedform? And to study the nonlinear characteristic of river dynamical process, the nonlinear river dynamic theoretical model is set up in part 2, the numerical system is stated for criterions and method of stability, and tropism characteristics as solution of river dynamic evolution; In part 3, as an application of this theory, the dynamical stability of the constant curvature river bend is calculated, obtaining the coherent vortex disturbance growth rate and the response range of perturbation wave number under different curvature; In addition, the nonlinear evolution of the river peristaltic process under a large-scale disturbance is discussed; Part 4 are some conclusions for river process.

2 Theory of nonlinear dynamic process

The nonlinear river dynamic process is to study the evolution of the river morphology, for a certain river subjected to disturbance, if the disturbance and the resulting response will eventually attenuate to zero, the river can be restored to its new equilibrium state, the river dynamic process is stable; otherwise, it is unstable. The generally mathematical expressions are shown in following section.

The factors to describe river dynamic process could be written in the form of vectors as $\vec{x}=(x_1,x_2,\cdots,x_n)$. The vector form of the generalized force is $\vec{f}=(f_1,f_2,\cdots,f_n)$. Hence, the river dynamic process can be described as follows:

$$L\vec{x}=\vec{f} \tag{1}$$

where L = general differential operator, rather than a linear operator of river dynamic process.

Assume a given initial state of a system $\vec{x}^0=(x_1{}^0,x_2{}^0,\cdots,x_n{}^0)$ with a disturbance $\vec{x}'=(x_1',x_2^1,\cdots x_n')<<\delta_0$. The stability analysis of river dynamic process is to investigate whether the disturbance is amplified or attenuated after some time periods (days or years). If, after a sufficiently long time, the disturbance $\vec{x}'=(x_1',x_2',\cdots,x_n')\to 0$, and the system restores to a new equilibrium state, the initial state is denoted as stable. In contrast, if the disturbance $\vec{x}'=(x_1',x_2',\cdots,x_n')\to\infty$, and the system cannot return to a new equilibrium state, the river system is defined as unstable.

2.1 Stability of river dynamic evolution

A particle with mass m and moving velocity v, subjected to external force $F=F(v)$, should satisfy the momentum theorem $m\dot{v}=F$. According to this equation, when the particle is under equilibrium state, $\dot{v}=0$, and therefore, $F(v)=0$. Following this analog idea, the singular point, or equilibrium state of the river system could be defined.

For the river dynamic process described in Eq. (1), assume that there is an equilibrium point $\vec{x}_0=(x_1^0,x_2^0,\cdots,x_n^0)$ with disturbance $\vec{x}'=(x_1',x_2',\cdots,x_n')$. Substituting $\vec{x}=\vec{x}_0+\vec{x}'$ into Eq. (1) yields:

$$L(\vec{x}_0+\vec{x}')=\vec{f} \tag{2}$$

Consideration of the equilibrium state leads to

$$L\vec{x}_0=\vec{f} \tag{3}$$

Subtract Eq. (3) from Eq. (2) and neglecting the terms with second or higher order gives the linear equation as:

$$L_0x'=Df(x_0)x' \tag{4}$$

where L_0 and Df_0 = linear parts of the operator L and the generalized force f, respectively. $\vec{x}_0$ is called the singular point of the river dynamic process, which includes any equilibrium state in river dynamic process.

2.2 Method for stability judging

Spectral stability criterion is used as a method for river dynamical process stability judging, the linear operator of the dynamic process Eq. (1) could be defined as $L'=L_0(x_0)-Df(x_0)$. Hence, Eq. (4) is re-arranged to follows:

$$[L_0(x_0)-Df(x_0)]x'=0 \tag{5}$$