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Original Research

Derivation and verification of minimum energy dissipation rate principle of fluid based on minimum entropy production rate principle

Guobin Xu [a,*], Lina Zhao [a], Chih Ted Yang [b]

[a] State Key Laboratory of Hydraulic Engineering Simulation and Safety, Tianjin University, China
[b] Civil and Environmental Engineering Department, Colorado State University, Fort Collins, US

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ABSTRACT

Based on non-equilibrium thermodynamics theory, minimum energy dissipation rate principle can be derived from minimum entropy production rate principle. Minimum entropy production rate principle is equivalent to minimum energy dissipation rate principle. In order to verify the fluid motion following minimum energy dissipation rate principle, RNG $k-\varepsilon$ turbulence model and GMO model of Flow 3D were applied to simulate fluid motion in a straight rectangular flume. The results show that fluid motion satisfies the minimum energy dissipation rate principle. The validity of minimum energy dissipation rate of alluvial rivers has been tested with field data. When a river system is at a relative equilibrium state, the value of its energy dissipation rate is at minimum. The minimum value depends on the constraints applied to the river system. However, due to the dynamic nature of a river, the minimum value may vary with respective to time and location. Minimum stream power and minimum unit stream power principles are special and simplified versions of the more general minimum energy dissipation rate principle.

1. Introduction

Helmholtz (1868) first proposed that in a body force field such as the gravity field, the energy dissipation rate of flow motion is less than any other motions with the same volume and velocity distribution when the inertia terms in the equations of motion for incompressible viscous flow are negligible. Velikanov (Великанов) applied minimum energy dissipation rate principle to fluvial dynamics, and explained the dynamic process of stream bed change as one of the three river bed theories (Hou, 1982). Yang and Song (1979, 1986), Yang (1994), and Chang (1979, 1984) made further advancements in the study of minimum energy dissipation rate principle and its applications.

Yang (1971, 1972) assumed that there is an analogy between a thermo and a river system. The concept of entropy was introduced by him to the study of river system. He believed that the only useful energy in the river system is its potential energy. He further assumed that the potential energy and elevation of a river system are equivalent to thermal energy and absolute temperature, respectively, of a heat system. Based on this analogy and direct application of entropy concept in thermodynamics, Yang showed that

$$\frac{dy}{dt}=\frac{dx}{dt}\cdot\frac{dy}{dx}=UJ=\text{a minimum} \tag{1}$$

where y=potential energy per unit weight of water in a river system, t=time, x=channel reach length, U=average flow velocity, and J=energy slope.

Yang (1972) and Yang and Song (1979) defined the UJ product as the unit stream power. Based on the concept of unit stream power, minimum energy dissipation rate principle can be expressed as:

$$\Phi=\gamma\iiint u_iJ_i\,\mathrm{d}x\mathrm{d}y\mathrm{d}z=\gamma QJ=\text{a minimum} \tag{2}$$

where Φ=energy dissipation rate per unit length, γ=specific weight of water=a constant, u_i=longitudinal local velocity, J_i=local energy slope, x, y, z=longitudinal, lateral and vertical coordinate, respectively, Q=average water discharge, and J=average energy slope. Eq. (2) can be simplified to

$$QJ=\text{a minimum} \tag{2a}$$

If u_i can be approximated by an average velocity U

$$UJ=\text{a minimum} \tag{2b}$$

* Corresponding author.
E-mail addresses: xuguob@sina.com (G. Xu), neimengguzhaolina@163.com (L. Zhao), ctyang@engr.colostate.edu, ctyang23@gmail.com (C.T. Yang).

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Eqs. (2a) and (2b) represent the minimum stream power (MSP) and minimum unit stream power (MUSP) principles, respectively. It is apparent that MSP and MUSP principles are special and simplified versions of the more general minimum energy dissipation rate (MEDR) principle.

This paper will show that minimum entropy production rate is equivalent to minimum energy dissipation rate according to non-equilibrium thermodynamics. The validity of MEDR or its special versions of MSP or MUSP will be tested with laboratory and field data.

2. Basic theory of non-equilibrium thermodynamics

The second law of thermodynamics states that all irreversible processes occurring in an isolated system always move towards the direction of increasing the entropy until the entropy reaches the maximum value. Therefore, the second law of thermodynamics is also called the principle of maximum entropy. The principle of maximum entropy was originally applied to isolated systems. However, some scholars believe that the principle of maximum entropy can also be applied to river systems, and considered that self-adjustments of alluvial rivers follow the principle of maximum entropy (Leopold & Langbein, 1962; Langbein, 1965; Qian et al., 1987; Deng, 1999). River systems are open systems, not isolated systems. Strictly speaking, principle of maximum entropy may not be applicable to open systems (Nicolis & Prigogine, 1977; Li, 1986; Xu & Lian, 2004). However, non-equilibrium thermodynamics theory was developed in classical thermodynamics applicable to open systems. Non-equilibrium thermodynamics theory states that in the linear region of non-equilibrium, evolution of an open system follows the principle of minimum entropy production rate.

For open systems, Prigogine divided the entropy change dS into the sum of two contributions (Nicolis & Prigogine, 1977), i.e.

$$\frac{\mathrm{d}S}{\mathrm{d}t}=\frac{\mathrm{d}_eS}{\mathrm{d}t}+\frac{\mathrm{d}_iS}{\mathrm{d}t} \tag{3}$$

where $\mathrm{d}_eS/\mathrm{d}t$ = entropy flux due to the exchanges of energy or matter with the environment, and $\mathrm{d}_iS/\mathrm{d}t$ = entropy production rate due to irreversible processes inside the system. The second law of thermodynamics states that $\mathrm{d}_iS/\mathrm{d}t$ is positive. If entropy is transported out of the system, $\mathrm{d}_eS/\mathrm{d}t<0$, otherwise $\mathrm{d}_eS/\mathrm{d}t>0$. The entropy flux may be positive, negative, or zero. The entropy change rate in an open system can be expressed as (Nicolis & Prigogine, 1977):

$$\frac{\mathrm{d}S}{\mathrm{d}t}=\iiint_V \sigma \mathrm{d}V-\iint_\Omega \boldsymbol{n}\cdot\boldsymbol{J}_s \mathrm{d}\Omega \tag{4}$$

where σ = local entropy production rate, representing the rate of entropy production per unit volume V; $\boldsymbol{n}$ = the outward unit vector to the surface element; $\boldsymbol{J}_s$ = entropy flux, representing the exchange rate of entropy through a unit of surface area Ω.

Comparing Eq. (3) with Eq. (4):

$$\frac{\mathrm{d}_iS}{\mathrm{d}t}=\iiint_V \sigma \mathrm{d}V \tag{5}$$

$$\frac{\mathrm{d}_eS}{\mathrm{d}t}=-\iint_\Omega \boldsymbol{n}\cdot\boldsymbol{J}_s \mathrm{d}\Omega \tag{6}$$

Nicolis and Prigogine (1977) made a series of complex derivation to yield:

$$\frac{\mathrm{d}P}{\mathrm{d}t}\le 0 \tag{7}$$

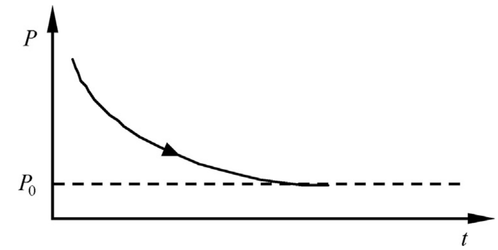

Fig. 1. Variation of entropy production rate in linear range.

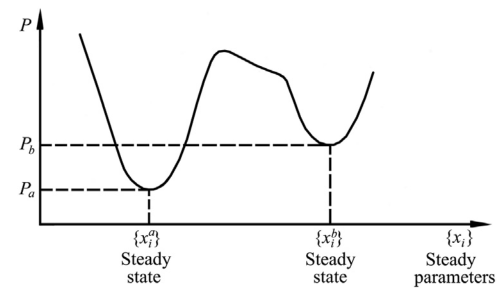

Fig. 2. Principle of minimum entropy production rate and stability of the steady state.

where P = total entropy production rate of a system in the form:

$$P=\iiint_V \sigma dV=\frac{\mathrm{d}_iS}{\mathrm{d}t} \tag{8}$$

If $\frac{dP}{dt}=0$, the system is at steady non-equilibrium state. If $\frac{dP}{dt}<0$, the system is away from steady non-equilibrium state. A system is either at equilibrium or non-equilibrium. If the state parameters of a system do not change with respective to time, the system is at steady state. Steady state is not necessarily equilibrium state, and it may also be at non-equilibrium state. Although the state parameters of a system do not change with respective to time at steady non-equilibrium states, macroscopic flow of physical property can still occur within the system. This is the difference between steady non-equilibrium and equilibrium states. Eq. (7) shows that, when boundary conditions are constant, in the linear range of the non-equilibrium state, the irreversible evolution processes inside an open system always move toward the direction of reducing the entropy production rate until its minimum value is attained. At equilibrium, the state of the system no longer varies with respective to time (see Fig. 1). At this point, the system is compatible with external constraints of steady non-equilibrium state. This conclusion is called the principle of minimum entropy production rate. The principle is one of the basic theories of non-equilibrium thermodynamics.

Prigogine derived the principle of minimum entropy production rate with a series of assumptions. These assumptions include the boundary conditions of the system is independent of time. The principle of minimum entropy production rate is applicable to any open system, regardless of the boundary condition to be constant or not. To reach this conclusion, we can examine the entropy change rate in Eq. (3) for open systems. If $\mathrm{d}S/\mathrm{d}t=\mathrm{d}_eS/\mathrm{d}t+\mathrm{d}_iS/\mathrm{d}t=0$, the system is at steady non-equilibrium state. If the external constraints (including boundary conditions) change, it will lead to the state of system parameters change. Then the system will deviate from the original steady state and evolve to a new steady state compatible with the new external constraints. In this process, the entropy production rate $P=\mathrm{d}_iS/\mathrm{d}t$ of the system is not necessarily monotonically decreasing. In other words, it may increase or decrease with the change of the entropy flux $\mathrm{d}_eS/\mathrm{d}t$. When the system evolves to a new steady state, the entropy generation rate must be the minimum value compatible with the new external constraints (see Fig. 2).