



Mathematical models for quantifying eruption velocity in degassing pipes based on exsolution of a single gas and simultaneous exsolution of multiple gases



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ABSTRACT

After the limnic eruptions at Nyos and Monoun in the 1980s, degassing pipes were installed to reduce the continuous increase of CO₂ at the bottom of these lakes. The degassing system consists of a vertical pipe from the lake bottom to the surface and a small pump located near the top of the pipe, which raises water in the pipe up to a level where it becomes saturated with gas, which in turn leads to volume expansion and eruption. This paper describes two new mathematical models for predicting eruption velocity in degassing pipes based on exsolution of a single gas and the simultaneous exsolution of multiple gases. The models were applied to the degassing system of lakes Nyos and Monoun, which contain two main gases, namely CO₂ and CH₄. Because the volume proportion of CH₄ is significant only in Lake Monoun, the Lake Nyos test case considered the CO₂ gas only, while as the Lake Monoun test case considered the simultaneous exsolution of CO₂ and CH₄. Good agreement between the results of the models and observed data is found for both test cases. The results for the eruption in Lake Monoun considering the two main gases measured in this lake (CO₂ and CH₄) were found to have a better agreement with the measurements compared to the model results obtained considering the main gas only (CO₂).

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1. Introduction

It has been shown that dissolved gas in a liquid is able to power violent eruptions (e.g., Woods, 1995; Zhang, 1996; Halbwachs et al., 2004; Zhang and Kling, 2006; Issa et al., 2014). Two types of gas-driven eruptions are known in nature, namely volcanic eruptions and lake eruptions. Volcanic eruptions are driven mainly by dissolved H₂O in magma at high temperatures (e.g., Wilson et al., 1980). Lake eruptions are driven mainly by exsolution of CO₂ from water at relatively low temperatures (Sigurdsson et al., 1987). The focus of this paper is on degassing pipes that are being used to reduce the probability of lake eruptions and hence our discussion is limited to this type of eruption. There are at least two known occurrences of lake eruptions, one in August of 1984 (Lake Monoun) and the other in August of 1986 (Lake Nyos), both in Cameroon, Africa. The eruption of Lake Monoun killed about 37 people, and the eruption of Lake Nyos killed about 1700 people up to 26 km away from the lake (Zhang and Kling, 2006).

During lake eruptions, large amounts of pressurized carbon dioxide, which were previously dissolved in the lower layers of the lakes, are released to the atmosphere (e.g., Sigurdsson et al., 1987; Halbwachs et al., 2004). These eruptions could be triggered by any external disturbance

or intrinsic instability of the lake, which could destabilize the density stratification of the lakes' water column (CO₂-laden water is denser than pure water), and hence release suffocating carbon dioxide (e.g., Halbwachs et al., 2004).

After the CO₂ eruption in lakes Nyos and Monoun in the 1980s, degassing pipes were installed in Lake Nyos in 2001 and in Lake Monoun in 2003 to reduce the continuous increase of CO₂ at the bottom of these lakes (Kusakabe et al., 2000; Halbwachs et al., 2004; Kling et al., 2005). The degassing system consists of a vertical pipe from the lake bottom to the surface (Halbwachs et al., 2004). A small pump located near the top of the pipe, raises the water in the pipe. As the water rises, the pressure decreases as a result both of the work done against gravity and pipe wall friction. The loss of pressure reduces the solubility of the dissolved gas in the moving liquid causing the formation of bubbles or gas exsolution. The exsolved gas causes rapid volume expansion, which accelerates the gas–liquid mixture upwards (foamy water), which in turn causes more gas exsolution due to the continued decompression. The foamy water is more or less a homogenous flow. Once the homogeneous foamy water breaks the water surface, an eruption is produced. Various photographs of these eruptions can be found in Halbwachs et al. (2004).

The volume expansion and eruption processes were verified in the laboratory (e.g., Zhang, 1997, 1998). In one of these experiments, water with high dissolved content of CO₂ in a test cell was suddenly

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decompressed, leading to CO₂ supersaturation, bubble nucleation and growth, volume expansion, and eruption (Zhang, 1997, 1998). These experiments showed that bubble nucleation is very fast, with a typical incubation time of a couple of milliseconds. The rapid bubble nucleation and bubble growth rates in the experiments may be used to argue that there is quasi-equilibrium between gas phase and liquid phase during geysering (e.g., Zhang and Kling, 2006).

Besides volcanic and lake eruptions, gas exsolution may also play a role in geysers occurring in stormwater and combined sewer systems. These systems may contain a mixture of toxic and non-toxic gases that can be present at varying levels depending upon the source (Hutter, 1993). The gases present in sewers may include ammonia (NH₃), hydrochloric acid (HCl), hydrogen sulfide (H₂S), methane (CH₄), carbon dioxide (CO₂), carbon monoxide (CO), sulfur dioxide (SO₂) and chlorine (Cl₂) (Viana et al., 2007; Hutter, 1993). For instance, ammonia, which is widely used in fertilizers, may be present in large amounts in stormwater and combined sewer systems. Some of the gases, such as ammonia, are easily absorbed in water. Ledig (1924) has shown that ammonia is absorbed in water at least 100 times faster than carbon dioxide. In most cases, it is likely that dissolved gases are below saturation, however they can be exsolved when the bottom water moves upward to the point where dissolved gas pressure exceeds local hydrostatic pressure.

This paper presents two mathematical models for predicting eruption velocity in degassing pipes based on exsolution of a single gas and the simultaneous exsolution of multiple gases. This paper is divided as follows. First, a mathematical model for quantifying eruption velocity in a degassing pipe based on exsolution of a single gas is presented. Second, the aforementioned model is extended for quantifying eruption velocity in a degassing pipe based on simultaneous exsolution of multiple gases. Third, the two models are applied to the degassing system of lakes Nyos and Monoun. Finally, the key results are summarized in the conclusion.

2. Mathematical model for quantifying eruption velocity in a degassing pipe based on exsolution of a single gas

The assumptions of this model are listed below:

- The bubbly two-phase flow in the vertical pipe is treated as a single-phase equivalent mixed flow.
- The liquid is assumed to be water with constant density (e.g., independent of pressure).
- The decompression process is assumed to be isothermal. This is justified because the decrease in temperature due to gas exsolution and bubble expansion is small, which is mainly due to the high heat capacity ratio of water (e.g., Zhang and Kling, 2006).
- There is exsolution of only one gas (e.g., CO₂). The second model addresses simultaneous exsolution of multiple gases.
- The gas is ideal, which is somewhat reasonable at low pressures.

The compressible Navier–Stokes equations for the vertical direction (z) assuming a one-dimensional flow can be written as (Shapiro, 1954)

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho w)}{\partial z} = 0 \quad (1)$$

$$\frac{\partial w}{\partial t} + \frac{1}{\rho} \frac{\partial p}{\partial z} + w \frac{\partial w}{\partial z} = -g - \frac{f}{2D} w^2 \quad (2)$$

where t is time, p is pressure, ρ is the density of the gas–liquid mixture, g is the gravitational acceleration, z is vertical distance measured from the point where the homogenous gas–liquid mixture starts to accelerate, w is the mixture rise velocity, f is the Darcy–Weisbach friction factor and D is the diameter of the vertical pipe. It is worth to mention that Shapiro (1954) uses the Fanning friction factor instead of the Darcy–Weisbach

friction factor. The Darcy–Weisbach friction factor is 4 times larger than the Fanning friction factor. Kim et al. (2015) using experiments with a carbonate mixture flow showed that the friction factor is independent of the fluid type.

Because the degassing is maintained over time, it is reasonable to assume that the degassing is a steady-state flow. Hence, the flow variables (e.g., ρ , p and w) are a function of z only. Thus, the time derivatives in Eqs. (1) and (2) are dropped and the partial derivatives are replaced by total derivatives. With these considerations, Eqs. (1) and (2) can be written as:

$$\rho dw + w d\rho = 0 \quad (3)$$

$$\frac{1}{\rho} \frac{dp}{dz} + w \frac{dw}{dz} = -g - \frac{f}{2D} w^2. \quad (4)$$

Combining Eqs. (3) and (4), the following equation is obtained:

$$\left(1 - \frac{w^2}{a^2}\right) \frac{1}{\rho} \frac{dp}{dz} = -g - \frac{f}{2D} w^2 \quad (5)$$

where a is the sound speed in the mixture flow given by $\sqrt{dp/d\rho}$. Note in Eq. (5) that dp/dz is negative and that both terms in the right side (gravity and friction) are negative. Hence, for Eq. (5) to be valid, the following condition needs to be satisfied.

$$w \leq a \quad (6)$$

This means that the maximum velocity of the gas–liquid mixture will be that of its sound speed. An equation for estimating the sound speed of a two-phase mixture is given by Mastin (1995)

$$a = \sqrt{\frac{k_l k_g}{\rho[(1-\phi)k_g + \phi k_l]}} \quad (7)$$

where ϕ is the volume fraction of gas (i.e., ratio of total volume of gas to total volume), k_l and k_g are the bulk modulus of liquid and gas, respectively. Measurements during the degassing of lakes Nyos and Monoun reported gas/water volume ratios ranging from 2.9 to 9 (Halbwachs et al., 2004). The latter indicates that at eruption the values of ϕ would range between 0.24 and 0.76. The bulk modulus of a material, which is the inverse of the compressibility, determines how much the material will compress under a given amount of external pressure. A representative value for the bulk modulus of water is 2.2×10^9 N/m² (2200 MPa). The bulk modulus for the gas depends on how the temperature varies during decompression. As discussed earlier, the temperature is kept more or less constant during the exsolution. For isothermal conditions, the bulk modulus of the gas (k_g) is equal to the gas pressure (Weast, 1984). At eruption, the pressure at the top of the degassing pipe will be near atmospheric (0.1 MPa), and hence the term $(1-\phi)k_g$ in Eq. (7) would range between 0.024 and 0.076 MPa and the term ϕk_l would range between 528 and 1672 MPa. Because $(1-\phi)k_g$ is a small fraction of ϕk_l (less than 0.01%), Eq. (7) can be reduced to

$$a \approx \sqrt{\frac{p_g}{\rho \phi}} \quad (8)$$

The density of the gas–liquid mixture is often expressed as $\rho = (1-\phi)\rho_l + \phi\rho_g$ (e.g., Zhang and Kling, 2006). As mentioned above, measured values of ϕ ranged between 0.24 and 0.76 (Halbwachs et al., 2004), ρ_l is about 1000 kg/m³ and ρ_g greatly depends on the pressure and temperature. At eruption, the pressure at the top of the degassing pipe will be near atmospheric. For atmospheric pressure and for temperatures higher than -20 °C, the air density will not exceed 1.4 kg/m³ (https://en.wikipedia.org/wiki/Density_of_air). With these considerations, at eruption, the minimum value of $(1-\phi)\rho_l$ would be about 240 kg/m³

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