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# An expansion–coalescence model to track gas bubble populations in magmas

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ABSTRACT

We propose a kinetic model that statistically describes the growth by decompression, exsolution and coalescence of a polydisperse population of gas bubbles in a silicate melt. The model is homogeneous in space and its main variable is a distribution function representing the probability to find a bubble of volume $v$ and mass $m$ at time $t$. The volume and mass growth rates are described by a simplification of the classical monodisperse bubble growth model. This simplification, which shortens computational time, removes the coupling between mass evolution and an advection–diffusion equation describing the behavior of the volatile concentration in the melt. We formulate three coalescence mechanisms: thinning of the inter-bubble planar films, film deformation by differential bubble pressure, and buoyancy-driven collision. Numerical simulations based on a semi-implicit numerical scheme show a good agreement between the coalescence-free runs and the monodisperse runs. When coalescence is introduced, numerical results show that coalescence kernels based on different physical mechanisms yield distinct evolutions of the size distributions. A preliminary comparison between runs and experimental data suggests a qualitative match of two out of the three proposed kernels. This kinetic model is thus a powerful tool that can help in assessing how bubble growth and coalescence occur in magmas.



## 1. Introduction

The remarkable link between the microphysics ruling gas bubbles that develop in magmas when they ascend in volcanic conduits and the large-scale dynamics of the resulting eruption is now firmly established. Bubble growth in viscous magmas is governed by decompression and exsolution of volatiles (mainly water) from the silicate melt into the bubbles themselves. This phenomena was first modeled by considering that the bubbles contained in a small volume of ascending magma are monodisperse, i.e. they all evolve in the same way and have the same radius, volume, mass, and pressure (Sparks, 1978). The most common model used since consists of a system of two ordinary differential equations describing the time evolution of the radius and mass of a single bubble that is coupled with an advection–diffusion equation modeling the space–time evolution of the volatile concentration in the melt surrounding the bubble (e.g., Toramaru, 1989, 1995; Proussevitch et al., 1993a; Proussevitch and Sahagian, 1998; Lyakhovsky et al., 1996; Navon et al., 1998; Lensky et al., 2001, 2004; Nishimura, 2004; Chouet et al., 2006; Ichihara, 2008; Forestier-Coste et al., 2012; Chernov et al., 2014).

Although this monodisperse bubble population model predicts porosities and bubble sizes that match well with experimental data (e.g., Lyakhovsky et al., 1996; Forestier-Coste et al., 2012), it assumes that bubbles are isolated and therefore does not take into account their interactions, such as the coalescence of two (or more) bubbles. The polydisperse nature of a bubble population causes differential growth (e.g., Larsen and Gardner, 2000; Gardner, 2009), which can enhance coalescence (Castro et al., 2012). Neglecting coalescence is a severe limitation because of its large impact on the bubble size distribution (e.g., Larsen et al., 2004; Burgisser and Gardner, 2005; Iacono Marziano et al., 2007; Martel and Iacono-Marziano, 2015) and because coalescence can create an interconnected network of bubbles from which the gas can escape (e.g., Saar and Manga, 1999; Burgisser and Gardner, 2005; Takeuchi et al., 2005; Gardner, 2007; Rust and Cashman, 2011; Bai et al., 2010). Gas escape from magma is arguably the most important consequence of bubble coalescence because it has long been recognized as a main control of the transition between effusive and explosive eruptions (e.g., Eichelberger et al., 1986; Jaupart and Allègre, 1991; Woods and Koyaguchi, 1994; Degruyter et al., 2012).

There have been efforts to overcome the monodisperse assumption in models. A first category of studies comprises phenomenological

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models that were aimed at reproducing natural size distributions (e.g., Blower, 2001; Blower et al., 2001; Gaonac'h et al., 1996, 2003, 2007). Very few studies populate the second category, which is composed of physical models (Lovejoy et al., 2004; L'Heureux, 2007). These physical models present a significant increment in complexity compared to monodisperse models. The model of L'Heureux (2007) considers the nucleation and growth of a polydisperse population of randomly located bubbles. The kinetic model of Lovejoy et al. (2004) describes the evolution of the distribution function of a set of bubbles that grow by decompression and coalescence. The distribution function represents the probability to find a bubble of a given volume at a specific time. The model is homogeneous in space. The volume growth rate is broadly defined as time dependent, and bubble coalescence is described by a Smoluchowski-type operator. Although it does not contain a numerical resolution of the full equation set, this work poses solid theoretical foundations to develop the next generation of model by providing solutions of simplified cases.

We propose a general kinetic model describing the evolution of a bubble population with vanishing velocities relative to the surrounding melt (i.e. valid in small magma volume where the population can be treated in a Lagrangian way). It is an extension of the kinetic model of Lovejoy et al. (2004) that includes growth mechanisms from the monodisperse model of Lensky et al. (2004) as formalized by Forestier-Coste et al. (2012) and coalescence mechanisms from Castro et al. (2012) and Lovejoy et al. (2004). The Methods and initial model section summarizes the monodisperse model that gives volume and mass evolution of each bubble class. The first section of the Results offers a simplification of this monodisperse model that avoids the coupling with the advection–diffusion equation and diminishes computational costs. A numerical study quantifies the error introduced by this simplification and the reader is referred to the Supplementary data to find B-growth, an open-source implementation of the monodisperse bubble growth model. We then propose suitable formulations of three coalescence operators. Since they depend on both the volume and the mass of the bubbles, these operators are best expressed in a two-dimensional form. Due to their complexity, two-dimensional coalescence operators have rarely been studied (Qamar and Warnecke, 2007a; Kumar et al., 2011). The abstract writing of the kinetic model follows in a rather classical way by means of Liouville's theorem and is in some sense a direct generalization of the Lovejoy et al. (2004) model. A summary of the discretization scheme is in Appendix A, and we use the numerical scheme developed in Forestier-Coste and Mancini (2012) for the coalescence kernel. Finally, we illustrate model capabilities by using a set of experimental data from Burgisser and Gardner (2005) and Castro et al. (2012).

## 2. Methods and initial model

### 2.1. Bubble size distributions of experimental samples

Numerical results are compared to four experimental samples described in Burgisser and Gardner (2005). The 2D analysis on thin sections reported in Burgisser and Gardner (2005) yielded a very small (~10) number of bubbles due to the restricted size of the 2D sections. Castro et al. (2012) re-analyzed some of these samples by Computed Tomography to increase the number of measured bubbles and improve the size distributions. We used the 3D reconstructed volume of sample PPE4 as described in Castro et al. (2012) and scanned samples PPE7, PPE10, and PPE11 using the same methodology so as to obtain 3D volumes of the samples with bubbles and melt separated in a binary fashion. De-coalescing was done by successively eroding the bubble network of one voxel-thick layer at a time with the ImageJ software (v. 1.47). De-coalesced bubble sizes were obtained by using the "Particle Analyzer" plugin of the BoneJ (version 1.3.11) bundle (Doube et al., 2010), from which equivalent diameters corrected for erosion numbers were calculated. The correction method used the measured surface area of each bubble multiplied by the number of erosions (generally 3). The size distribution of PPE4 obtained using this method was very similar to that obtained by manually separating bubbles with the Blob3D software (Castro et al., 2012).

### 2.2. Monodisperse population modeling

In a viscous, crystal-free magma, the growth of a pre-existing monodisperse population of bubbles, i.e. of a set of bubbles of identical size at any given time and not interacting, can be described by giving the time, $t$, evolution of the radius, $R$, and mass, $M$, of one single bubble and assuming that it is surrounded by a spherical influence domain (i.e. the region of melt exchanging water with the bubble) of radius $S$ (Proussevitch et al., 1993a; Fig. 1). Variables are summarized in Table 1. Subscripts 0 indicate initial values, diacritic dot indicates time differentiation. Diacritic tilde marks dimensional variables while its absence indicates dimensionless variables (hence $t$, $R$, $M$, and $S$ are dimensionless). All the equations listed in the main text are given in dimensionless form, and Table S1 in the Supplementary data lists their dimensional counterparts. We use the mathematical formulation by Forestier-Coste et al. (2012) of the physical model proposed by Lensky et al. (2004). Briefly, these works describe the evolution of bubble radius and mass as a function of decompression and exsolution by the following dimensionless system of ordinary differential equations:

$$\dot{R} = \frac{R}{\eta \Theta_V}\left(P - P_a - \frac{\Sigma}{R}\right) \quad (1)$$

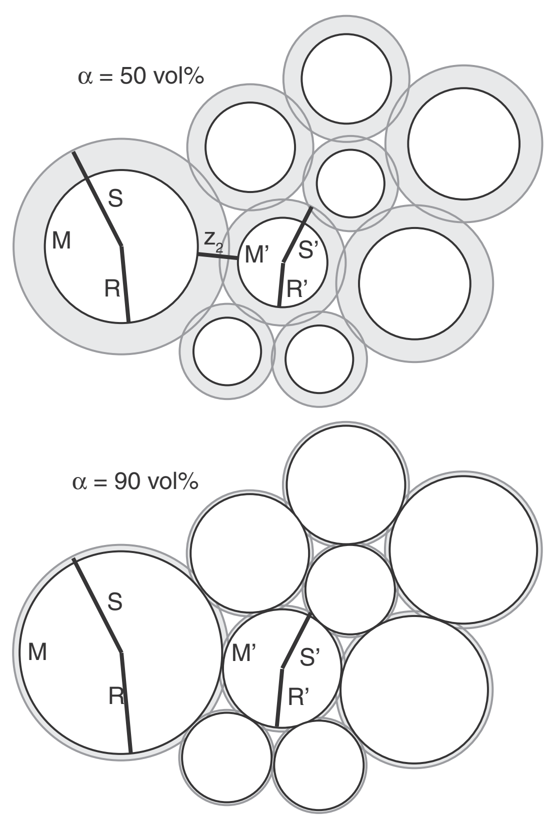


**Fig. 1.** Schematics of polydisperse arrangements of gas bubbles of radius $R$ and mass $M$ (white) with their surrounding melt shell of radius $S$ (gray) that slightly overlap. Primes indicate the second bubble in a bubble pair considered by coalescence mechanisms, and $z_1$ is the distance between two bubbles in a pair. A) 50 vol.% porosity ($\alpha$). B) 90 vol.% porosity ($\alpha$). Drawings are scaled so as to represent 2D illustrations, albeit the models consider 3D spherical bubbles. Porosity is thus equal to $R^2/S^2$ ($R^3/S^3$ in 3D) and B) represents the maximum packing situation occurring at ~90 vol.% (60–70 vol.% in 3D, see Appendix B).