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Sensitivity analysis of a one-dimensional model of a volcanic plume with particle fallout and collapse behavior

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ABSTRACT

We run a volcanic plume model with uncertain boundary conditions and entrainment related model parameters. Output variables tested for their sensitivity to the inputs are total rise height, and mass flux of particles into the umbrella cloud or downwind plume. Boundary or source conditions are vent radius, initial velocity, grain size mean and grain size standard deviation. Model parameters are entrainment rate, α, wind entrainment rate, β, and wind speed.

Five sensitivity metrics were considered. Three of these are calculated for each given point in the input parameter space, by perturbing the input variable around fixed points. Two global sensitivity measures quantify the impact on the output of the input over its entire uncertain domain.

We find that vent radius and initial speed have a much more profound effect on both outputs than does total grain size distribution. Plume rise height and particle mass flux are sensitive to the entrainment parameters, α and β, but these parameters are not of greater importance than the wind speed. This suggests that while efforts to better characterize entrainment parameters through laboratory experiments is important, similar efforts should be made to collect appropriate meteorological data for the region near the site of the eruption.

1. Introduction

One-dimensional, numerical eruption column or plume models (Costa et al., this volume) have found a use in estimation of the amount of ash emplaced into the atmosphere at an estimated plume height (e.g., Folch et al., 2008). The nature of the sensitivity of the output ash loading or plume height to the variables and parameters that are incorporated in a given plume model is however poorly known (Scollo et al., 2008; Degruyter and Bonadonna, 2012; Woodhouse et al., 2015, this volume). One expects, based on previous experience, that plume height and atmospheric loading should primarily be functions of grain size, vent radius, and plume velocity (Sparks et al., 1997).

The eruption plume model discussed in the present contribution was introduced by Bursik (2001). As part of a larger program of improvement and recasting, in the present contribution, it has been modified in a number of ways. These changes were precipitated by the work presented in Bursik et al. (2012) and Stefanescu et al. (2014). The changes to the original plume model are as follows. The model:

1. Has been modified to provide input to PUFF or HYSPLIT (Bursik et al., 2013).
2. Can use radiosonde or NWP data directly to get atmospheric parameters.
3. Can estimate atmosphere above the top of radiosonde or NWP data used as input.
4. Can be run in stochastic mode with uncertain inputs of volcanic boundary conditions as well as entrainment parameters and wind speed.
5. Can be run in inverse mode to estimate source parameters.
6. Can simulate collapse behavior, to allow fountain height to be recorded. In these cases, there is no injection of pyroclasts from the vent into the atmosphere.
7. Includes a refined model for plume rise height calculation.

Other changes to the model not included in the present version – to keep it as close to the model of Bursik (2001) as possible – are the following:

1. Modules for water have been added (Glaze and Baloga, 1996).
2. Double-precision and adaptive step-size now used.
3. Previously little-documented, optional, umbrella cloud and fallout modules (Bursik et al., 2009) are available.

The model was originally unnamed, but was later called BENT (Bursik et al., 2009). The present incarnation is called “puffin,” to

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emphasize the fact that it can be used to provided input to PUFF or HYSPLIT (Bursik et al., 2013).

We concentrate herein on discussing results from running the model with uncertain boundary conditions and entrainment related parameters in a full sensitivity analysis. The output variables to be tested for their sensitivity to the inputs are total rise height, H_T, and mass flux of particles into the umbrella cloud or downwind plume, $\dot{m}_p|_{H_B}$, which are responsible for the atmospheric loading. First we introduce the current set of equations of motion, and then we introduce the methods and metrics to be used in the analysis. Finally, we present a consideration of the meaning of the results.

2. Model of plume motion

The deterministic plume model was presented previously (Bursik, 2001), and is an integral, one-dimensional model for plumes that entrain mass and momentum from the wind (Hewett et al., 1971; Wright, 1984). The model includes a spectrum of pyroclasts of different grain sizes and settling speeds that move at the same speed as the plume gases until falling out from the plume margins. Once falling they can be re-entrained. It is a trajectory model, and therefore well-suited for adaptation to and coordination with meteorological models and data. The following presentation follows that in (Bursik, 2001), but has been explicitly modified to highlight the variables and parameters that are now treated as stochastic.

2.1. Coordinate system

In the following analysis, the downwind distance is x, z is up, and s represents the distance along the plume axis from the vent. Theta, ϑ, is the inclination of the plume centerline to the horizon. The equations expressing the relationship between (x,z) and (s,ϑ) are then given by:

$$x = \int \cos\vartheta ds, \tag{1}$$

$$z = \int \sin\vartheta ds. \tag{2}$$

2.2. Equations of plume motion

In plumes that are significantly affected by the wind, the entrainment speed, U_ε, must be a function of wind speed, $V(\xi)$, now a stochastic variable given as a function of the unit random variable, ξ, as well as axial plume speed, U. A number of wind entrainment relationships have been investigated (see Table 2.1 in Wright, 1984, for an older summary). Reasonable correspondence between one such entrainment relation and experimental data has been obtained (Hewett et al., 1971):

$$U_\in = \alpha(\xi)|U - V(\xi)\cos\vartheta| + \beta(\xi)|V(\xi)\sin\vartheta|, \tag{3}$$

where $\alpha(\xi)$, the radial entrainment parameter and $\beta(\xi)$, the wind-entrainment parameter, are both now stochastic – but constant – parameters. Thus, we now have three stochastic *model parameters*: $V(\xi), \alpha(\xi), \beta(\xi)$. With this knowledge, henceforth the ξ will be dropped from the description of these variables. Eq. (3) assumes that the magnitude of the horizontal wind component is much larger than the vertical component. The practical meaning behind V, α and β being stochastic is that numerous, carefully selected values for these will be substituted into the equations of motion. Among other things, this will allow for exploration of the range of values for the entrainment parameters from the literature. Some of these different values may arise from near-vent phenomena, where plume density may be five times that of the ambient atmosphere (Sparks et al., 1997), and plume decompression occurs in the crater (Woods and Bower, 1995), or from a Richardson number dependence (Wang and Law, 2002; Kaminski and Tait, 2005).

For mass conservation (continuity) of the plume, we have:

$$\frac{d}{ds}\left(\pi b^2 \rho U\right) = 2\pi\rho_a b U_\epsilon + \sum_{i=1}^{N} \frac{dM_i}{ds}, \tag{4}$$

where b is the characteristic plume radius, ρ is the bulk plume density, ρ_a is the ambient atmospheric bulk density, and M_i represents the mass flux of pyroclasts of size fraction i within the plume. The first term on the right-hand side represents the gain in mass flux by entrainment of air, whereas the second term represents the loss of mass flux by fallout of pyroclasts.

The conservation of mass flux of particles for multiple grain size fractions, M_i, is given by (Ernst et al., 1996):

$$\frac{dM_i}{ds} = -\frac{\hat{p} w_s}{bU} M_i, \tag{5}$$

where $\hat{p}$ is a probability that an individual particle will fall from the plume and w_s is the settling speed of a particle in the given size class (in the current model, $i = 1$ to 19 for pyroclasts between 10 and -8 Φ at $1-$ Φ intervals). The probability of fallout, $\hat{p}$, is a function of plume geometry and re-entrainment (Bursik, 2001), and should have an approximately constant value of ~0.23 with no re-entrainment, based on the geometry of plume margins in a quiescent atmosphere (Ernst et al., 1996). Because of the strong inflow towards the plume caused by entrainment, pyroclasts <~10 cm are, however, re-entrained at lower heights in a plume after falling from greater heights. Fitting a curve through experimental results for a vertical plume in a quiescent ambient Ernst et al. (1996), a reasonable, purely heuristic, form of the re-entrainment function, f, is:

$$f = 0.43\left(1 + \left[\frac{0.78}{F_0^{1/2}\mu_0^{-1/4}/w_{si}}\right]^6\right)^{-1}, \tag{6}$$

where F_0 is the specific thermal flux at the vent, $F_0 = b_0^2 U_0 C_{v\,0} T_0$, and μ_0 is the specific momentum flux at the vent, given by $\mu_0 = b_0^2 U_0^2$, and settling speeds of pyroclasts, $w_{s\,i}$ is calculated as a function of height, given atmospheric density and viscosity. With wind, Eq. (3) can be at best a poor approximation, as the pyroclasts on the downwind side would often not be re-entrained, given that the net horizontal wind speed can be away from the plume. However, at low wind speed, a zeroth-order assumption – made herein – is that the enhanced fallout on the downwind side is balanced by an enhanced re-entrainment on the upwind side.

The equation for conservation of axial momentum is:

$$\frac{d}{ds}\left(\pi b^2 \rho U^2\right) = \pi b^2 \Delta\rho g \sin\vartheta + V\cos\vartheta \frac{d}{ds}\left(\pi b^2 \rho U,\right), \tag{7}$$

where the first term on the right-hand side represents the change in momentum caused by the component of gravitational acceleration, $\Delta\rho g = (\rho_a - \rho)g$, in the axial direction, and the second term represents entrainment of momentum from wind. Note that this equation is modified from that in Bursik (2001), by taking out an explicit dependence on dM_i/ds, which because the effect of loss of pyroclasts on momentum flux is already counted in the second term on the RHS, resulted in a doubling of the effect of pyroclast fallout on plume dynamics. The conservation of the radial component of momentum is given by:

$$\left(\pi b^2 \rho U^2\right)\frac{d\vartheta}{ds} = \pi b^2 \Delta\rho g \cos\vartheta - V\sin\vartheta \frac{d}{ds}\left(\pi b^2 \rho U\right), \tag{8}$$