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Rheological flow laws for multiphase magmas: An empirical approach

Mattia Pistone [a,*], Benoît Cordonnier [b], Peter Ulmer [c], Luca Caricchi [d]

[a] Department of Mineral Sciences, National Museum of Natural History, Smithsonian Institution, 10th Street & Constitution Avenue NW, 20560 Washington, DC, United States
[b] Piazza Duca degli Abruzzi 20, 65124 Pescara, Italy
[c] Department of Earth Sciences, ETH-Zurich, Clausiusstrasse 25, CH 8092, Zurich, Switzerland
[d] Department of Earth Sciences, University of Geneva, Rue des Maraîchers 13, CH 1205, Geneva, Switzerland

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ABSTRACT

The physical properties of magmas play a fundamental role in controlling the eruptive dynamics of volcanoes. Magmas are multiphase mixtures of crystals and gas bubbles suspended in a silicate melt and, to date, no flow laws describe their rheological behaviour. In this study we present a set of equations quantifying the flow of high-viscosity (>10^5 Pa·s) silica-rich multiphase magmas, containing both crystals (24–65 vol.%) and gas bubbles (9–12 vol.%). Flow laws were obtained using deformation experiments performed at high temperature (673–1023 K) and pressure (200–250 MPa) over a range of strain-rates ($5 \cdot 10^{-6}$ s^{-1} to $4 \cdot 10^{-3}$ s^{-1}), conditions that are relevant for volcanic conduit processes of silica-rich systems ranging from crystal-rich lava domes to crystal-poor obsidian flows. We propose flow laws in which stress exponent, activation energy, and pre-exponential factor depend on a parameter that includes the volume fraction of weak phases (i.e. melt and gas bubbles) present in the magma. The bubble volume fraction has opposing effects depending on the relative crystal volume fraction: at low crystallinity bubble deformation generates gas connectivity and permeability pathways, whereas at high crystallinity bubbles do not connect and act as "lubricant" objects during strain localisation within shear bands. We show that such difference in the evolution of texture is mainly controlled by the strain-rate (i.e. the local stress within shear bands) at which the experiments are performed, and affect the empirical parameters used for the flow laws. At low crystallinity (<44 vol.%) we observe an increase of viscosity with increasing strain-rate, while at high crystallinity (>44 vol.%) the viscosity decreases with increasing strain-rate. Because these behaviours are also associated with modifications of sample textures during the experiment and, thus, are not purely the result of different deformation rates, we refer to "apparent shear-thickening" and "apparent shear-thinning" for the behaviours observed at low and high crystallinity, respectively. At low crystallinity, increasing deformation rate favours the transfer of gas bubbles in regions of high strain localisation, which, in turn, leads to outgassing and the observed increase of viscosity with increasing strain-rate. At high crystallinity gas bubbles remain trapped within crystals and no outgassing occurs, leading to strain localisation in melt-rich shear bands and to a decrease of viscosity with increasing strain-rate, behaviour observed also in crystal-bearing suspensions.

Increasing the volume fraction of weak phases induces limited variation of the stress exponent and pre-exponential factor in both apparent shear-thickening and apparent shear-thinning regimes; conversely, the activation energy is strongly dependent on gas bubble and melt volume fractions. A transient rheology from apparent shear-thickening to apparent shear-thinning behaviour is observed for a crystallinity of 44 vol.%. The proposed equations can be implemented in numerical models dealing with the flow of crystal- and bubble-bearing magmas. We present results of analytical simulations showing the effect of the rheology of three-phase magmas on conduit flow dynamics, and show that limited bubble volumes (<10 vol.%) lead to strain localisation at the conduit margins during the ascent of crystal-rich lava domes and crystal-poor obsidian flows.

1. Introduction

Magma viscosity is recognised as one of the critical factors controlling the viscous versus brittle modality of magma ascent, emplacement, and, finally, the style of volcanic eruptions (e.g. Gonnermann and Manga, 2007). The viscosity is thus a key parameter in modelling igneous petrological and volcanological processes. During the last four decades significant effort has been addressed to characterise and model the viscosity of silicate melts of various compositions (e.g. Hess and Dingwell, 1996; Giordano et al., 2008; Robert et al., 2013). However, magmas are natural suspensions that generally contain gas bubbles, solid crystals and silicate melt. Several studies focused on the physical properties of crystal-bearing, bubble-free systems (Lejeune and Richet,

* Corresponding author.
E-mail address: PistoneM@si.edu (M. Pistone).

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1995; Deubener and Brückner, 1997; Smith, 1997; Bagdassarov and Dorfman, 1998a, 1998b; Paterson, 2001; Petford, 2003; Mecklenburgh and Rutter, 2003; Rutter and Neumann, 1995; Rutter et al., 2006; Lavallée et al., 2007, 2008; Caricchi et al., 2007, 2008; Champallier et al., 2008; Cordonnier et al., 2009, 2012; Kohlstedt and Holtzman, 2009; Mueller et al., 2010, 2011; Cimarelli et al., 2011; Vona et al., 2011; Forien et al., 2011; Del Gaudio, 2014; Moitra and Gonnermann, 2015) or bubble-bearing, crystal-free systems (Bagdassarov and Dingwell, 1992, 1993a, 1993b; Lejeune et al., 1999; Llewellin et al., 2002; Okumura et al., 2006, 2008, 2009, 2010; Rust and Manga, 2002a, 2002b; Stein and Spera, 1992, 2002; Kameda et al., 2008; Caricchi et al., 2011). These studies have largely contributed to understand the influence of solid crystals *or* gas bubbles on magma rheology and the results were summarised in two main models for two-phase magmatic systems: (i) the model of Costa et al. (2009) for crystal-bearing systems; and (ii) the model of Llewellin et al. (2002) for bubble-bearing systems.

(i) The model of Costa et al. (2009) provides a set of empirical equations describing the relative viscosity (i.e. ratio between viscosity of the multiphase system and suspending fluid viscosity) of crystal-bearing magmas as function of crystal volume fraction and strain-rate, including the effect of crystal geometrical characteristics (i.e. crystal shape). The model is calibrated on suspension characterised by a wide range of crystallinity (30 to 80 vol.%) and strain-rates (10^{-7} and $10^{-0.5}$ s^{-1}).
(ii) The equations presented by Llewellin et al. (2002) describe the rheology of bubble-bearing suspensions. The model accounts for the contrasting effect of bubbles on the rheology of magmas at high and low capillary number (*Ca*). Gas bubbles behave as rigid inclusions for $Ca < 1$, and inviscid objects at $Ca > 1$.

Recently, Mader et al. (2013) presented a detailed review of the rheology of two-phase magmas, with *either* crystals *or* bubbles. They propose constitutive equations based on the analogue data of Mueller et al. (2010) (for crystal-bearing systems) and Llewellin et al. (2002) (for bubble-bearing systems). Their model is only applied to some of the rheological datasets of crystal-bearing basaltic systems (Ishibashi, 2009; Vona et al., 2011) and cannot be extended to high-viscosity (>10^5 Pa s) and crystal-rich (>40 vol.%) magmatic systems.

To date, only a few rheological models account for the simultaneous presence of particles and bubbles. Harris and Allen (2008), following the approach of Phan-Thien and Pham (1997), adopted the treatment for the viscosity of a three-phase mixture to describe the rheology of multiphase basaltic lava flows. Their model is formulated for cases in which one phase (crystals *or* bubbles) has a much larger size than the other. If one phase is much smaller than the other in relative size, that phase is treated as an effective medium along with the melt viscosity; therefore, the three-phase mixture is reduced to a two-phase system in the rheological model. When crystals are much smaller than bubbles, the three-phase mixture becomes the proxy of a bubble-bearing suspension; when crystals are much larger than bubbles, the three-phase mixture becomes the proxy of a crystal-bearing suspension. When both phases display similar size, the resulting multiphase mixture viscosity is intermediate between the two above-described cases. However, this intermediate rheological scenario is not discussed in detail in the work of Harris and Allen (2008). This model was calibrated with degassed (0.1 to 0.4 wt.% H_2O) basaltic lava flow case studies from Mauna Loa (Hawaii) and Mount Etna (Italy) where the bulk viscosities of entire suspensions composed of crystals, vesicles (i.e. no gas-pressurised bubbles) and melt in different volumetric proportions range from 110 to 12.5 10^3 Pa s. Thus, the range of bulk viscosity is significantly different from that of silica-rich compositions, where melt viscosity alone is higher than 10^5 Pa s in the range of temperature of 973–1273 K and H_2O contents of 0.1 to 2.5 wt.% (Giordano et al., 2008). Therefore, the model of Harris and Allen (2008) might not be directly applied to describe the rheology of high-viscosity systems containing gas-pressurised bubbles.

Recently, Truby et al. (2015) built a three-phase rheology model on two-phase constitutive equations by using the effective medium theory in which the bubble suspension is treated as a continuous medium that suspends the particles. This approach carries the implicit assumption that the bubbles are small compared with the particles (similarly to the case when bubbles are smaller than crystals, as reported by Harris and Allen, 2008). Truby et al. (2015) validated the model against experimental data for three-phase suspensions of bubbles and spherical particles in the low-capillarity regime (in which flow is steady and bubbles remain spherical/rigid during deformation) and applied it to basaltic systems characterised by low melt viscosity (<10^5 Pa s). Despite the potential of their model to be applied to a wide range of crystallinity (<50 vol.%) and bubble content (>30 vol.%), this three-phase rheological model does not include the dynamics of mutual interactions between crystals and gas bubbles occurring during deformation. Moreover, due to the application of the effective medium theory, in their two-phase type model bubbles are not deformable in a low-viscosity suspension. For these reasons, this model might not be appropriate to quantify the rheology of high-viscosity systems deforming in unsteady flow conditions, which are characteristic for the ascent of highly viscous magmas (e.g. Caricchi et al., 2011).

No comprehensive rheological models for multiphase high-viscosity silica-rich systems exist. Multiphase suspensions are characterised by local spatial arrangements of different phases (i.e. gas bubbles, crystals, and melt) during deformation, depending on the respective volume fractions, sizes, and interactions of phases, and stress/strain-rate partitioning within the microstructural domain of the suspension. Textural arrangements of crystals and/or bubbles (e.g. crystal alignment, bubble elongation, etc.) are strongly sensitive to strain-rate dependent processes. Hence, the local decrease or increase of the viscosity is attributed to shear-thinning or shear-thickening behaviour respectively. However, the definition of shear-thinning and -thickening implies a non-Newtonian behaviour, passing through the origin of a stress/strain-rate plot, but also recoverable with time once the system goes back to its initial conditions. If this definition is valid for most cases, there are however a few phenomenological cases, such as crystal size reduction and outgassing, where the system is not able to return to the original rheological configuration. For such cases, *pseudo* or *apparent* shear-thinning and *apparent* shear-thickening may be more appropriate terms to use for the description of these rheological behaviours. Bagdassarov et al. (1994) performed oscillatory experiments at atmospheric pressure on a synthetic rhyolite containing crystals and bubbles (equal contents of 8, 16, and 40 vol.%) to determine the rheological behaviour of three-phase mixtures in the temperature range from 1023 to 1323 K. However, the restricted range of parameters investigated in this study limits the possibility of using these results to calibrate rheological models for three-phase magmas. Avard and Whittington (2012); Kendrick et al. (2013); Lavallée et al. (2007, 2008, 2013); Vona et al. (2013), and Ashwell et al. (2015) conducted similar high-temperature compression tests on natural rock samples with variable crystal content (28–65 vol.%) and porosity (5–52 vol.%). In these dominantly fracture-free experiments isolated and connected porosity does not contain pressurised gas; thus, sample rheology is mostly driven by sample compaction (i.e. porosity collapse at <45 vol.% crystals and/or >25 vol.% connected porosity). Therefore, the effect of open pores (containing no pressurised gas) cannot be easily compared with that of gas-pressurised bubbles in a multiphase magma. Recently, Pistone et al. (2012) investigated a wide range of crystallinity (24 to 65 vol.%) and limited bubble content (9 to 12 vol.%), showing that the addition of relatively small bubble volume fractions to particle-bearing suspensions significantly decreases magma viscosity (Pistone et al., 2012, 2013). As an example, 9–10 vol.% bubbles at high crystallinity (55–