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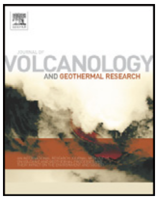

Influence of grain-size distribution on the dynamics of underexpanded volcanic jets

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ABSTRACT

The dynamics of underexpanded volcanic jets is studied by means of a Eulerian multiphase flow model, accounting for kinetic and thermal non-equilibrium between gas and particles. Two-dimensional numerical simulations are analysed in terms of the scaling properties of the gas–solid flow based on the particle Stokes number *St*, defined as the ratio of the particle relaxation time τ_s and a typical flow time scale, here taken as the formation time of the jet Mach disk τ_M. For monodisperse mixtures, fine particles with $St \ll 1$ are tightly coupled to the gas during decompression and the typical flow patterns of supersonic free jets are reproduced. The multiphase flow model, in this case, gives results comparable to a dusty-gas model. However, for $St \gg 1$ we demonstrate that the effects of particle inertia and gas–particle drag are so relevant that the shock wave structure associated with gas decompression is obliterated, although the mixture velocity remains supersonic. We are therefore able to identify two different regimes of jet decompression (with/without Mach disk) that appear to be associated exclusively with the size of the ejected particles. For bidisperse mixtures, multiphase flow numerical simulations show that the features of underexpanded jet depend on the relative mass load of fine and coarse particles. A scaling analysis has been performed by means of a hybrid multiphase flow model, in which fine particles are mixed with the gas (in a dusty-gas approximation), while coarse particles are modelled as a disperse phase, with a modified Stokes number accounting for the mixture properties of the carrier pseudofluid. Both scaling analysis and numerical simulations with the hybrid model corroborate the analysis based on the Stokes number for bidisperse underexpanded jets, although particle–particle momentum exchange between particles of different size can affect the multiphase flow dynamics, especially in regimes with $St \sim 1$. We finally extend our analysis to polydisperse mixtures, by defining a threshold between fine and coarse particles on the basis of their Stokes number. Numerical simulations and scaling analysis based on the hybrid model (in which all fine particles are mixed with the gas in the dusty-gas approximation and the coarsest particles are described by a single hydraulically equivalent disperse phase) demonstrate that the total particle size distribution can have a significant effect on the jet decompression dynamics, potentially affecting the stability and large-scale behaviour of the volcanic column.

1. Introduction

Explosive volcanic eruptions are characterized by the injection in the atmosphere of a polydisperse mixture of gases and mostly solid particles at high velocity and temperature. Evidence of higher-than-atmospheric pressure at the conduit exit is provided by empirical and theoretical analysis of the shape of volcanic jets and resulting craters (Woods, 1988; Woods and Bower, 1995) as well as from infrasonic and acoustic measurements (Morrissey and Chouet, 1997; Johnson, 2003; Fee and Matoza, 2013; Fee et al., 2013). As a consequence, the erupting mixture must equilibrate to the ambient pressure in the atmosphere through gas decompression and expansion.

The decompression of the gas–particle mixture is a complex non-linear process, which is affected, interalia, by the conduit flow and fragmentation dynamics (Wilson et al., 1980; Papale et al., 1998) and by the shape of the volcanic crater (Woods and Bower, 1995; Koyaguchi et al., 2010; Ogden, 2011). In any case, when sonic conditions are reached at the conduit exit, the underexpanded flow can manifest the features of supersonic flows above the vent. Such supersonic stage has been hypothesized for explosive Plinian and Vulcanian eruptions (Kieffer and Sturtevant, 1984; Woods and Bower, 1995; Ogden et al., 2008b; Orescanin et al., 2010) and for volcanic blasts (Kieffer, 1981). However, imaging of the internal structure of a volcanic flow is still challenging, so that the supersonic jet structure must be inferred from external observations or from theoretical/computational considerations.

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A first, theoretical understanding of the dynamics of supersonic volcanic jets has been achieved by approximating the gas–pyroclast mixture as a homogeneous dusty-gas (Marble, 1970), i.e., by assuming kinetic and thermal equilibrium between gas and particles (Kieffer, 1984; Woods and Bower, 1995; Ogden et al., 2008b; Orescanin et al., 2010). Under such hypotheses, the eruptive mixture would behave as a dusty-gas with average density given by the sum of the gas and particle bulk densities and with average rheological properties. In particular, due to the thermal behaviour of particles, the isoentropic coefficient γ would be around 1 (i.e., corresponding to almost isothermal adiabatic transformations) and the mixture sound speed would be significantly lowered with respect to pure gas (Kieffer, 1984).

As a result, volcanic jet dynamics would closely resemble that of supersonic nozzles, resulting in the rapid expansion and acceleration of the fluid to high Mach numbers (Ladenburg et al., 1949; Lewis and Carlson, 1964; Orescanin et al., 2010). A series of expansion waves form at the vent exit (Prandtl–Meyer expansion), which are reflected as compression waves at the jet flow boundary. In a narrow crater (with opening angle narrower than about 30) oblique shocks reflect on the jet axis forming a regular structure. In wide craters or for free expansion, the compression waves coalesce to form a barrel shock and a standing normal shock wave (Mach disk). Through the Mach disk, the fluid is rapidly compressed and decelerated to subsonic speeds. Above the Mach disk, the fluid moves in subsonic regime in the core of the jet, surrounded by a supersonic shell separated by a tangential discontinuity (slip-line), eventually torn by turbulence.

As pointed out by Ogden et al. (2008b) and Saffaraval et al. (2012), the resulting jet structure can strongly influence the stability and large-scale behaviour of the volcanic column, affecting the entrainment and the mixing processes which drive the potential transition to a positive or negative buoyancy regime (convective plume or collapsing column).

Despite the early recognition of the Mach disk structure in multiphase flow simulation of eruption columns (Dobran et al., 1993; Pelanti and LeVeque, 2006; Esposti Ongaro et al., 2008), the effect of gas–particle and particle–particle non-equilibrium processes on volcanic jet decompression dynamics has never been addressed specifically. However, the equilibrium assumption is valid only on a restrict range of particle sizes. In this work, we adopt scaling analysis and a Eulerian, multiphase flow model to investigate the behaviour of supersonic vertical volcanic jets affected by non-equilibrium processes between the gaseous and the particulate phases. Our present work is based on a preliminary benchmark study (Carcano et al., 2013) which, together with similar previous analyses (Dartevelle, 2007; Ogden et al., 2008a) demonstrates the "empirical adequacy" (Oreskes et al., 1994) of the adopted computational fluid dynamic models for the study of volcanic jet dynamics.

2. Scaling analysis

In underexpanded gas–particle jets, velocity disequilibrium between gas and particles may occur in the rapid expansion above the vent, where the gas phase accelerates and is subject to adiabatic cooling, and across the Mach disk, where the gas velocity is reduced abruptly and heated across the compressive shock. Here, we analyse non-equilibrium effects by means of the characteristic time scales of the multiphase jet, i.e. the particle relaxation time τ_s and the Mach disk formation time τ_{Ma}. We thus introduce the Stokes number for the multiphase underexpanded jet as the ratio between the two characteristic times:

$$St = \frac{\tau_s}{\tau_{Ma}}. \tag{1}$$

2.1. Particle relaxation time

The (velocity) relaxation time τ_s of a particle moving with speed $\mathbf{v}_s$ in a surrounding fluid, which is moving with velocity $\mathbf{v}_g$, can be estimated from the particle momentum equation (Eq. (A.2b)) by assuming that all the forces acting on the particle are negligible except for the drag force, that is

$$\frac{\partial}{\partial t}(\epsilon_s \rho_s \mathbf{v}_s) \simeq Dg, s\left(\mathbf{v}_g - \mathbf{v}_s\right). \tag{2}$$

Following Marble (1970), Burgisser and Bergantz (2002) and Dufek and Bergantz (2007), we define the particle relaxation time as

$$\tau_s = \frac{\epsilon_s \rho_s}{D_{g,s}}. \tag{3}$$

The solution of Eq. (2) is approximated as $\mathbf{v}_s = \mathbf{v}_g$ after a time about $t \approx 5\tau_s$, when the difference between the two velocities reaches a value less than 1% of the initial one.

For dilute mixtures and low gas–particle Reynolds number, as defined in Eq. (A.13), the particle relaxation time can be computed from Eqs. (3) and (A.11), obtaining

$$\tau_s \simeq \frac{\rho_s d_s^2}{18\mu_g} \tag{4}$$

where ρ_s and d_s are the particle density and diameter and μ_g is the dynamic viscosity of the carrier fluid phase. For gas–particle Re > 1000 this expression should be corrected (Carcano et al., 2013) resulting in $\tau_s \simeq \frac{\rho_s d_s^2}{0.33 \mathrm{Re} \mu}$, i.e. lower than the value for low-Re regimes.

We can apply the same reasoning to evaluate a thermal relaxation time, starting from the enthalpy equation for particles (Eq. (A.3b)), where all terms except that of interphase heat exchange (Eq. (A.15)) are neglected. The thermal relaxation time is thus defined as:

$$\tau_{Ts} = \frac{\epsilon_s \rho_s}{Q_{g,s}}. \tag{5}$$

By taking Eq. (A.17a) for the heat exchange coefficient and assuming Nu ~ 2 for relative Reynolds number ≪ 200 (Dobran et al., 1993), we obtain:

$$\tau_{\mathrm{Ts}} \simeq \frac{3}{2} \frac{\mu_g c_{p,g}}{k_g} \frac{\rho_s d_s^2}{18\mu_g} = \frac{3}{2} \frac{\mu_g c_{p,g}}{k_g} \tau_s = \left(\frac{3}{2}\,\mathrm{Pr}\right) \tau_s \tag{6}$$

where the Prandtl number Pr is defined by Eq. (A.17c). For water vapour at high temperature, the Prandtl number is of order 1, so that the particle thermal relaxation time is about the same order of magnitude as the velocity relaxation time. We will therefore base our analysis and discussion on the velocity relaxation time τ_s only.

2.2. Mach disk formation time

The supersonic jet process is characterized by different time scales. Following Orescanin et al. (2010), we will assume the formation time of the Mach disk shock τ_{Ma} to characterize the dynamics of supersonic jets. Assuming steady, sonic conditions at the vent, τ_{Ma} represents the time needed by the first rarefaction wave issuing from the vent rim to reach the jet centreline.

The Mach disk formation time has been estimated by Orescanin et al. (2010) by means of geometric considerations. Denoting c the speed of shock waves in air and $v = |\mathbf{v}_{ps}|$ the mixture velocity at the vent, at