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Local method of approximate particular solutions for two-dimensional unsteady Burgers' equations

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ABSTRACT

The local method of approximate particular solutions (LMAPS) is first proposed to solve Burgers' equations. In avoiding the ill-conditioning problem, the weight coefficients of linear combination with respect to the function values and its derivatives can be obtained by solving low-order linear systems within the local supporting domain. Then the global solutions can be obtained by reformulating the local matrix in the global and sparse matrix. The obtained large sparse linear systems are directly solved instead of using more complicated iterative method. The numerical experiments have shown that the proposed method is suitable for solving the two-dimensional unsteady Burgers' equations with high accuracy and efficiency.

1. Introduction

Burgers' equation is a useful model for many interesting physical problems, such as shock wave, acoustic transmission, traffic and aerofoil flow theory, turbulence and supersonic flow as well as a prerequisite to the Navier–Stokes equations. The problems modelled by the Burgers' equation can be considered as an evolutionary process in which a convective phenomenon is in contrast with a diffusive phenomenon. Most of the existing numerical methods were reported to be able to successfully solve Burgers' equations, such as the finite difference method (FDM), finite element method (FEM) and boundary element method (BEM). The mesh-free methods developed in the past two decades have attracted the attention of researchers for its ability to handle scattered data without using any mesh. Examples are the multiquadrics (MQ) method [1–3], the meshless local Petrov–Galerkin (MLPG) method [4,5], the method of fundamental solutions (MFS) [6–10] and the differential quadrature (DQ) [11–13]. Among them, Hon and Mao [1] applied the MQ method to the one-dimensional unsteady Burgers' equation, while Li et al. [2] used the MQ method to solve two-dimensional problems. The (MFS) is also successfully applied to solving linear and non-linear PDEs such as the linear diffusion equation, Poisson's equation [6], biharmonic equation [7] and Stokes flow [8]. Due to the existence of the convective term in Burgers' equations, the MFS cannot be used directly. But this can be dealt with by the Eulerian–Lagrangian method (ELM) [9,10].

Based on the concepts of particular solutions and radial basis functions (RBFs), the method of approximate particular solutions (MAPS) has been developed for solving PDEs [14–16] for its simplicity and effectiveness. Typically, a global method is constructed by using a certain number of collocation points in the entire domain which generally results in a full and dense matrix. However, the resultant matrix often suffers from near singular, dense and ill-conditioned problems which hinder the application of RBF-based methods to solve large-scale problems. Some novel techniques have been proposed to circumvent

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this issue such as domain decomposition [17], the improved truncated singular value decomposition [18], the localized formulations [19–21], fast multipole expansion techniques [22], etc. Among these new techniques, the local scheme based on local support interpolations appears to be more efficient in handling a large number of collocation points.

The aim of this paper is to demonstrate the capability and formulation of the novel localized method of approximate particular solutions (LMAPS) for solving the unsteady nonlinear two-dimensional Burgers' equations. The outline of the remaining sections is as follows. In Section 2, the formulations of the LMAPS are presented for solving Burgers' equations by applying localized formulations within local supporting points in detail. Numerical tests are given in Section 3 to demonstrate the effectiveness of the proposed simulation procedure and conclusions are drawn in Section 4.

2. Formulation of the LMAPS for incompressible Burgers' equations

2.1. Governing equations

We consider the two dimensional unsteady Burgers' system with two variables,

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{Re}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right), \tag{1}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \frac{1}{Re}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right), \tag{2}$$

subject to the initial conditions:

$$\begin{aligned} u(x, y, t_0) &= \varphi_1(x, y) \quad (x, y) \in \Omega \\ v(x, y, t_0) &= \varphi_2(x, y) \quad (x, y) \in \Omega \end{aligned} \tag{3}$$

and boundary conditions:

$$\begin{aligned} \mathcal{B}u(x, y, t) &= \varphi_3(x, y, t) \quad (x, y) \in \partial\Omega \\ \mathcal{B}v(x, y, t) &= \varphi_4(x, y, t) \quad (x, y) \in \partial\Omega \end{aligned} \tag{4}$$

where Re is the Reynolds number, $\mathcal{B}$ is a boundary differential operator, u and v denote the components of velocity in x and y directions.

2.2. Local approximation of derivatives

The ill-conditioning of the interpolation matrix and the large scale dense matrix has limited the development of global RBF-based meshless methods. To mitigate these difficulties, local schemes based on the local supporting domain seem to be more efficient methods, less sensitive to changes on the value of the shape parameter and avoiding the ill-conditioning problem.

We denote $\{\mathbf{x_j}\}_{j=1}^{n_s}$ as the interpolation points inside the influence domain. For any point $\mathbf{x}_p$, its supporting domain Ω_p contains the nearest n_s interpolation points $\{\mathbf{x}_j\}_{j=1}^{n_s}$ to $\mathbf{x}_p$. The value of a function $f(\mathbf{x_p})$ can be approximated by a linear combination of n_s function values in the following form:

$$f(\mathbf{x}_p) \simeq \hat{f}(\mathbf{x}_p) = \sum_{j=1}^{n_s} a_j \Phi\left(\|\mathbf{x}_p - \mathbf{x}_j\|\right), \quad \mathbf{x}_p \in \Omega_p \tag{5}$$

where Ω_p is the local supporting domain and

$$\Delta\Phi\left(\left\|\mathbf{x} - \mathbf{x}_j\right\|\right) = \phi\left(\left\|\mathbf{x} - \mathbf{x}_j\right\|\right), \tag{6}$$

where $\|.\|$ is the Euclidean norm and $\phi(\|\mathbf{x}-\mathbf{x}_j\|)$ are RBFs. We prefer to choose multiquadric (MQ) RBFs that have been found to be able to provide very accurate approximations in most of the applications and have been widely used by researchers. The normalized MQ-RBFs [12] can be written as

$$\phi(\|\mathbf{x} - \mathbf{x}_j\|) = \sqrt{\left(\bar{x} - \frac{x_j}{R_i}\right)^2 + \left(\bar{y} - \frac{x_j}{R_i}\right)^2 + \bar{c}^2}, \tag{7}$$

where $\bar{x} = \frac{x}{R_j}, \bar{y} = \frac{y}{R_j}$, (x, y) represents the coordinates of a point in the supporting region in the physical space, $(\bar{x}, \bar{y})$ denotes the coordinates in the unit circle, and R_j is the radius of the minimal circle enclosing all nodes in the supporting region for the node $\mathbf{x}_j$. Note that the shape parameter c for traditional MQ-RBFs is equivalent to $\bar{c}R_j$. The value of c is crucial to the accuracy of the approximation. The errors and the condition number of a matrix depend on the number of nodes in the