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Soliton solutions and chaotic motions for the (2 + 1)-dimensional Zakharov equations in a laser-induced plasma

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ABSTRACT

The (2 + 1)-dimensional Zakharov equations arising from the propagation of a laser beam in a plasma are studied in this paper. Analytic soliton solutions are obtained by means of the symbolic computation, based on which we find that $|E|$ is inversely related to ω_{pe}, but positively related to m_i and c_s, while n is inversely related to ω_{pe} and ω_L, but positively related to n_0, with E as the envelope of the high-frequency electric field, n as the plasma density, while ω_{pe}, ω_L, n_0, m_i and c_s as the plasma electronic frequency, frequency of the laser beam, mean density of the plasma, mass of an ion and ion-sound velocity in the plasma, respectively. Head-on interaction is found to be transformed into an overtaking one with ω_{pe} increasing or n_0 decreasing. Also, period of the bound-state interaction decreases with ω_L decreasing. Considering the driving forces in the laser-induced plasma, we explore the associated chaotic motions as well as the effects of ω_L, ω_{pe}, k_L, n_0, m_i, c_s, ω_{F_1} and ω_{F_2}, where k_L is the wave number of the laser beam, ω_{F_1} and ω_{F_1} represent the frequencies of driving forces, respectively. It is found that the chaotic motions can be weakened with ω_{pe}, c_s and ω_{F_1} increasing, or with n_0, m_i and ω_{F_2} decreasing, and the periodic motion can occur when ω_{F_1} reaches the critical value 2π, while the chaotic motions are independent of ω_L and k_L.

1. Introduction

When a high-power laser beam is focused onto a solid target, the target absorbs the laser energy, and then after such target melts and evaporates, a "laser-induced plasma" forms at the laser-focusing area [1]. Studies on the laser-induced plasmas and nonlinear phenomena in a laser-induced plasma have been carried out [2–4]. Numerical codes which couple the low-frequency hydrodynamics of the plasma with the propagating laser light have been developed [5]. Interaction between the laser beam and a plasma has been studied, and relativistic electron transport in the dense matter inside the target has been investigated [6]. Plasma waves driven by the multiple laser pulses and nonlinear regions of the interaction between the plasma waves and laser pulses have been studied [7].

For the applications, people have proposed the laser-induced plasma accelerators, which are the techniques for accelerating such charged particles as the electrons, positrons and ions, including the laser wakefield accelerator, plasma beat wave accelerator and self-modulated laser wakefield accelerator [7,8]. A laser-induced plasma accelerator of only a centimeter's length has been found to produce the 1 GeV electron bunches which are characterized by the $\geq 100pC$ charge

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at the ∼100 MeV mean energy with the energy spread approximately few percent and divergence approximately a few milliradians, and those electron bunches have been applied as the front-end injectors for conventional accelerators or the drivers for short-pulse radiation sources [8,9].

To describe the propagation of a laser beam in a plasma, people have derived the (2 + 1)-dimensional Zakharov equations [10–12]:

$$iE_t + i\frac{k_L\omega_L}{c^2}E_y + \frac{c^2}{2\omega_L}E_{xx} - \frac{\omega_{pe}^2}{2n_0\omega_L}nE = 0, \tag{1a}$$

$$n_{tt} - c_s^2 n_{xx} - \frac{\omega_{pe}^2}{4\pi m_i c^2}|E|_{xx}^2 = 0, \tag{1b}$$

where E, a complex function of x, y and t, denotes the envelope of the electric field, n, a real function of x, y and t, represents the plasma density measured from its equilibrium value, y is the direction of the propagation of the laser beam, x refers to the direction that is transversal to the beam propagation, t is the normalized time, ω_L and k_L represent the frequency of the laser beam and its wave number, respectively, c is the speed of light in vacuum, ω_{pe} is the plasma electronic frequency, n_0 is the mean density of the plasma, m_i represents the mass of an ion, and c_s is the ion-sound velocity in the plasma [10–12]. It has been demonstrated that Eqs. (1) are locally well posed in the whole space [10]. Behaviors of the laser beam in the plasma have been studied [11]. Through the discussions on Eqs. (1), transverse motions of the laser beam in a plasma have been studied experimentally [12]. Exp-function method and first integral method have both been employed on Eqs. (1), traveling and periodic waves have been found, and the properties of two-dimensional coherent structures have been discussed [13–17].

On the other hand, it has been found that even small differences in the initial conditions, such as the rounding errors in the numerical computation or perturbations in the background, yield the diverging outcomes, resulting in the long-term prediction impossible [18,19]. Studies have pointed out that such nonlinear phenomena as the chaos and turbulence have been found in the Zakharov–Kuznetsov (ZK) [20,21], sine-Gordon (SG) [22,23], Korteweg–de Vries (KdV) [24,25] and derivative nonlinear Schrödinger (dNLS) [26–28] equations when the perturbations are taken into consideration [29–33]. Period-doubling sequences of the KdV and SG equations [18,19], quasi-period-doubling sequences of the ZK and dNLS equations [34,35], frequency-locking of the Chen system [31–33] and intermittency of the Rossler and Chen systems [29,30] have been introduced. In addition, effects of the forces such as the random noises, radially-symmetric azimuthal force and periodic force in the plasmas have been studied numerically and experimentally [20,21,31–33,36].

Beyond Eqs. (1), we plan to study the following equations [12,7,36]:

$$iE_t + i\frac{k_L}{c^2}\omega_L E_y + \frac{c^2}{2\omega_L}E_{xx} - \frac{\omega_{pe}^2}{2n_0\omega_L}nE = \alpha_1 \sin\omega_{F_1}t, \tag{2a}$$

$$n_{tt} - c_s^2 n_{xx} - \frac{\omega_{pe}^2}{4\pi m_i c^2}|E|_{xx}^2 = \alpha_2 \sin\omega_{F_2}t, \tag{2b}$$

which arise in a laser-induced plasma affected by the driving forces $\alpha_1 \sin\omega_{F_1}t$ and $\alpha_2 \sin\omega_{F_2}t$, with α_1 and α_2 as the driving amplitudes, ω_{F_1} and ω_{F_2} as the driving frequencies, and F_1 and F_2 labeling the different forces [12,7,36]. Note that the difference between Eqs. (1) and (2) roots in the driving forces. Experimental studies on Eqs. (2) have been done with the plasma wavelength $\lambda_p \approx 33$ μm, mean density of the plasma $n_0 = 10^{18}$ cm^{-3}, and a 0.5 mJ, 4 ps laser pulse [7]. Stability analysis on Eqs. (2) has been carried out [12]. Analytic solutions and the associated nonlinear waves for Eqs. (2) have been obtained [36]. Generation of the bursting patterns in the Duffing oscillator with time-delayed feedback, including the symmetric fold, fold bursting and symmetric Hopf bursting when the periodic forcing changes slowly, has been investigated, and conditions of the fold bifurcation and Hopf bifurcation as well as its stability related to the external forcing and delay have been calculated [37–40].

In this paper, we will extend the work in Refs. [10–12,7,36], with the aim of exploring the nonlinear phenomena in the laser-induced plasma described by Eqs. (1) or Eqs. (2), e.g., the soliton propagation and interaction, effects of the plasma parameters on the solitons, chaotic motions as well as the effects of the driving forces and plasma parameters on the chaos. Thus, by means of the symbolic computation and Hirota method [41–45], we will derive the soliton solutions of Eqs. (1) in Section 2. In Section 3, the soliton propagation and interaction, including the effects of the plasma parameters (e.g., ω_{pe}, ω_L and n_0) on the solitons, will be discussed. Introducing the driving forces, in Section 4, we will study the associated chaotic motions of Eqs. (2). With the power spectra and phase projections investigated, our emphasis will be paid on the effects of the driving forces and plasma parameters on the chaos, as well as the transitions among the different chaotic motions. Finally, the conclusions will be in Sec. 5.