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An amplitude finite element formulation for electromagnetic radiation and scattering

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ABSTRACT

Electromagnetic radiation and scattering in an exterior domain within the context of a finite element method has traditionally involved imposing a suitable absorbing boundary condition on the truncation boundary of the numerical domain to inhibit reflection from it. In this work, based on the Wilcox asymptotic expansion of the electric far-field, we propose an amplitude formulation within the framework of the nodal finite element method, whereby the highly oscillatory radial part of the field is separated out a-priori so that the standard Lagrange interpolation functions that are used have to capture a relatively gently varying function. Since these elements can be used in the immediate vicinity of the radiator or scatterer (with few exceptions which we enumerate), it is more effective compared to methods that impose absorbing boundary conditions at the truncation boundary, especially for high-frequency problems. The proposed method is based on the standard Galerkin finite element formulation, and uses standard Lagrange interpolation functions, standard Gaussian quadrature and the same degrees of freedom as a conventional formulation. We show the effectiveness of the proposed formulation on a wide variety of radiation and scattering problems involving both conducting and dielectric bodies, and involving both convex and non-convex domains with sharp corners.

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1. Introduction

The traditional approach to solving electromagnetic radiation and scattering problems on unbounded domains within the context of a finite element method has been to truncate the domain at some finite distance, and imposing some suitable absorbing boundary condition (ABC) on the truncation boundary so as to reduce the reflection of the outgoing waves from it. Results using the most simple (first-order) ABC, known as Sommerfeld radiation condition, are presented in [1]. Some second and higher-order ABCs have been presented in [1–4]. A modified form of the second-order ABC ('Joly–Mercier ABC') has been presented in [5], and has been applied to the three-dimensional Maxwell equations in [6]. Another approach to model the Sommerfeld radiation condition accurately at the truncation boundary has been to combine the Boundary Integral method (BI) with the standard finite element method [7–15]. In these works, the interior domain is modeled with edge-based finite elements and the truncation surface with BI, with field continuity imposed between the two. Although BI terminations are more accurate compared to the conventional ABCs, a major drawback of this method is the fully populated matrix for the BI block, resulting in increased memory requirements and computational time.

A relatively recent approach introduced by Berenger is the concept of a 'perfectly matched layer' [16–23], where a 'lossy material' which surrounds the domain is used to damp out the propagating wave. Another recent approach to

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model the non-reflecting boundary conditions is the use of infinite elements [24–26], which the authors have shown to be more efficient compared to a conventional formulation based on an ABC. Conventional elements are required between the radiator/scatterer and the infinite elements in most of the infinite element formulations, while the proposed amplitude formulation can be used in the full domain provided the origin is not part of the domain. Special shape functions are required for the infinite elements, while in the proposed formulation standard Lagrange interpolation functions are used. A comparison of the performance is carried out in Section 5.4.

In most of the existing literature, electromagnetic radiation and scattering problems have been solved using edge elements, for example, see [1–28]. These elements impose only tangential continuity of the primary field variable across interelement boundaries, whereas the normal component is allowed to be discontinuous. There are relatively very few works that use nodal finite elements either for interior or exterior electromagnetic problems, see for example, [29–37]. While the use of nodal finite elements is especially desirable while solving multiphysics problems where structural or fluid flow variables need to be coupled with the electromagnetic field variables, the main difficulty has been in modeling sharp corners and edges where a regularized formulation (which is generally used to eliminate the spurious modes) is unable to model the singular fields correctly [1,29,32]. Otin and coworkers [33–35] have suggested a remedy to this problem, namely, that of using a zero penalty in regions near to the singularity. We use a slightly modified version of this strategy in our work in conjunction with the new proposed strategy that we outline below for solving scattering problems involving nonconvex domains with sharp corners.

In the current work, we propose an amplitude formulation that is based on the Wilcox asymptotic expansion of the electric field at large distances from the radiator/scatterer [38,39]. Although a similar idea was used in [40–42] in the context of acoustical problems, the presence of singularities, material discontinuities and the existence of spurious modes in the current problem (to name just a few of the additional complications) makes an extension of that strategy to the current problem nontrivial. A similar idea in the electromagnetics context was also proposed in [43]. However, they restrict their formulation to two-dimensional problems, do not consider scatterers with nonconvex boundaries/sharp corners, and require the solution of an equation for the phase. So in some sense, the current work may be considered as a generalization of their work to solve more realistic problems. If r denotes the radial coordinate of a spherical coordinate system, and k the wave number, the main idea in the amplitude formulation is to a-priori separate out the highly oscillatory e^{-ikr} part from the interpolation function so that the finite element interpolation function has to capture only a relatively gently varying function which it does quite effectively, as we will demonstrate with several examples where the solutions obtained using the proposed strategy are compared with analytical solutions and existing numerical strategies. The superior performance of the proposed amplitude formulation over a conventional strategy that uses an absorbing boundary condition is more evident at higher frequencies. Furthermore, unlike the PML where new elements are introduced in a 'lossy' layer surrounding conventional elements, the AMP elements (elements based on the proposed amplitude formulation) can be used in the immediate vicinity of the radiator/scatterer (except when the origin is part of the domain, in which case, we use a combination of conventional elements and AMP elements with the conventional elements used near to the origin); following this principle, all the numerical examples in Section 5 which deal with radiation and scattering from conductors use only AMP elements, while those dealing with dielectrics use a combination of conventional and AMP elements.

One aspect that we would like to emphasize at this stage is that although the amplitude formulation have been developed within the context of a nodal finite element formulation in this work (since our goal is to ultimately couple it with structural or fluid mechanics variables where nodal finite elements, which have a very different data structure compared to edge elements, are generally used), the same ideas can be used to develop amplitude formulation in edge elements. Thus, there is no intrinsic dependence of the amplitude formulation on the nodal finite element framework.

The outline of the remainder of the paper is as follows. In Sections 2–4, we develop the variational formulation and the finite element formulations for the conventional and the AMP elements; the formulations for both radiation and scattering from conductors and dielectrics are presented. Section 5 presents a series of examples demonstrating the superior performance of the amplitude formulation over the conventional strategy that uses an absorbing boundary condition by comparing the solutions obtained against either analytical solutions or other numerical results such as edge element results obtained using HFSS [44]. Besides considering scattering from 'smooth' bodies such as spheres or ellipsoids, we also consider examples where the scattering body has sharp corners such as cubes or circular cylinders, where, as is well-known, the (unmodified) regularized nodal finite element method fails.

2. Mathematical formulation

Under harmonic excitation, the governing differential equation is given as

$$\nabla \times \left(\frac{1}{\mu} \nabla \times \boldsymbol{E} \right) - \frac{k^2}{\mu} \boldsymbol{E} = -i\omega \boldsymbol{j}, \tag{1}$$

where $\boldsymbol{E}$ is the electric field, μ is the magnetic permeability, $i = \sqrt{-1}$, $\boldsymbol{j}$ is the current density, $k = k_0\sqrt{\mu_r \epsilon_r}$ is the wave number of the medium, $k_0 = \omega^2/c^2$ is the wave number of vacuum, ω is the frequency of excitation, $c = 1/\sqrt{\epsilon_0 \mu_0}$ is the speed of light in vacuum, $\epsilon_r = \epsilon/\epsilon_0$ and $\mu_r = \mu/\mu_0$ are the relative permittivity and relative permeability, and ϵ_0 and μ_0

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