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Integration of Thiele Continued Fractions and the method of fundamental solutions for solving unconfined seepage problems

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ABSTRACT

Unconfined seepage through an earth dam or a levee is recognized as a challenging problem. This complexity is mainly due to the fact that determination of the phreatic line through the dam/levee body is not straightforward. For simulation purposes, mesh generation as well as accurate and smooth alteration of the phreatic line at the junction with the downstream slope (referred to as exit point) for an unconfined seepage problem with complex geometry generally makes cumbersome numerical solutions. This study presents an innovative boundary-type meshfree method to determine the phreatic line location in unconfined seepage problems. The current study explicitly addresses the problem that alternative methods commonly face to deal with the exit point. The method is developed based upon integrating the Method of Fundamental Solutions (MFS), Particle Swarm Optimization (PSO) algorithm, and Thiele Continued Fractions (TCF). To accurately estimate the phreatic line location, the proposed framework uses MFS to solve the flow continuity equation, TCF to generate the phreatic line and PSO to optimize the phreatic line location generated by TCF. As a boundary method, MFS only deals with the boundaries of the domain and consequently, it only takes the exact position of phreatic line as a variable boundary. The proposed approach employs TCF to guarantee that the phreatic line is tangent to the downstream slope at the exit point, a characteristics which is important especially for the cases where abrupt changes occur in the phreatic line near the exit point. For comparison and validation purposes, the phreatic lines determined by the proposed approach for two unconfined seepage problems are compared and verified against those obtained from alternative analytical and numerical methods as well as a set of experimental results. An excellent agreement is demonstrated upon comparison of the proposed method to the results attained from the analytical solutions and experimental tests.

1. Introduction

Unconfined seepage through an earth dam or a levee is recognized as a challenging problem [1,2]. Determination of pore pressures, the quantity of seepage passing through the body and foundation, and the locus of the phreatic line are critical parameters that can significantly impact the performance of earth dams and levees. The flow continuity equation is used as

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governing equation for seepage problems. This equation can be solved using analytical and numerical approaches. However, analytical approaches are applicable for simple geometries and several limitations are reported in literature for the use of analytical methods for solving problems with complex geometry [3,4]. The position of the phreatic line cannot easily be obtained and advanced numerical methods such as the Finite Element Method (FEM) have been broadly used in solving unconfined seepage problems [5,6]. Accuracy and convergence of mesh-based methods such as FEM strongly depend on the mesh generation technique which is used in analyses. Cividini and Gioda [7] showed that standard FEM algorithms may result in an inaccurate and noisy phreatic line in the vicinity of a pervious boundary exposed to the atmosphere. Convergence of the conventional mesh-based algorithms can also suffer from potential mesh distortion and subsequent singularity that may occur due to the dynamic geometry of the phreatic line during the solution process. Though, improved mesh-based methods such as FEM [8] and the Finite Volume Method (FVM) [9] have been developed using fixed or adaptive grids or a combination of both to overcome the aforementioned drawbacks.

Along with recent advances in computational mechanics within the last a few decades, a new family of numerical methods called meshfree methods has been introduced and extensively utilized in engineering and applied mathematics [10–15]. Meshfree methods can provide a promising alternative for solving problems which involve iterative procedures without facing difficulties associated with the mesh generation step [15,16]. Meshfree methods use a set of scattered nodes within the problem domain and boundaries (i.e., domain-type meshfree) or only within the domain boundaries (i.e., boundary-type meshfree) to simulate the problem without discretization [17]. For interpolation among the scattered nodes, the Radial Basis Function (RBF) method is used as primary tool to approximate the results in both boundary- and domain-type meshfree methods [18,19]. The Local RBF based on the Differential Quadrature (RBF-DQ) method is a domain-type meshfree method which has recently been used in seepage analyses [20]. The radius of supporting domain, however, is constant for all nodes and it requires special consideration along the boundaries. Since RBF-DQ is sensitive to the node locations, the constant radius of the supporting domain can cause some shortfalls in complicated geometries. Moreover, a large number of nodes are required in domain type meshfree methods because each single RBF does not satisfy the governing equation [21,22]. Therefore, the computational cost increases in domain-type meshfree methods. Alternatively, the Natural Element Method (NEM) can be considered as it does not depend on the nodes location. However, NEM is not a fully meshfree method since it relies upon Delaunay triangulation [15,23]. As a remedy for the above-discussed drawbacks and to improve the efficiency of computational efforts, The Method of Fundamental Solutions (MFS) is introduced to solve boundary value problems with moving geometries [24,25]. Chaiyo et al. [26] used MFS, a meshfree boundary-type method, for solving unconfined seepage problems. Although the location of phreatic line was determined, the imposition of the phreatic line where the seepage path exposes to the atmospheric pressure was not still easy especially where the slope of downstream is vertical. This condition can be referred to as smooth alteration. Shahrokhabadi and Ahmadi [27] coupled MFS with the Particle Swarm Optimization (PSO) algorithm but their solution still faced some difficulties in satisfying the smooth alteration condition at sharp boundaries.

This paper presents a new boundary-type meshfree approach to determine the phreatic line location in unconfined seepage problems. The current study explicitly addresses the problem that alternative methods commonly deal with the junction of the phreatic line with the downstream slope of the domain (also referred to as exit point). At the exit point, the phreatic line should be tangent to the downstream slope. This property is a consequence of the fact that the fluid pressure must be atmospheric both along the phreatic line and along the downstream slope of the domain. The proposed approach is developed based upon integrating MFS, PSO, and Thiele Continued Fractions (TCF). Employing TCF distinguishes the current study from the previous works [23,27]. The proposed framework uses MFS to solve the flow continuity equation, TCF to generate the phreatic line and PSO to optimize the phreatic line location generated by TCF. PSO is used to investigate the fitness of the phreatic line location in the solution procedure. The fitness of the results is introduced as a constraint which should be considered in the solution steps. Additional constraints are the phreatic line inclination at the exit point, energy principles, and the value of pressure head on the phreatic line. For comparison and validation purposes, the phreatic lines determined by the proposed approach for two unconfined seepage problems (a rectangular domain and a trapezoidal domain) are compared and verified against those obtained from alternative numerical and analytical methods as well as a set of experimental results.

2. Governing equation and boundary conditions

The governing equation of fluid flow in porous media can be obtained by solving the equation of continuity. The governing equation reduces to Laplace's equation for laminar steady-state flow in a saturated, isotropic and homogeneous domain. Darcy's law is used to establish the relationship between the total head, ϕ, and flow velocity. As defined, ϕ is the algebraic summation of the pressure head ($\frac{p_w}{\rho g}$, p_w is the pore pressure, ρ is the density of water, and g is the gravitational acceleration) and the elevation head (y) in any position within the domain (i.e., $\phi = \frac{p_w}{\rho g} + y$). As shown in Fig. 1, the boundary conditions can be summarized as follows [28]:

1. Impermeable boundary (Line OA): Flow velocity orthogonal to this boundary is zero (i.e., $\frac{\partial \phi}{\partial n_v} = 0$, n_v is the normal vector on the impermeable boundary). The impermeable boundary defines streamlines.