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Using simple plume models to refine the source mass flux of volcanic eruptions according to atmospheric conditions

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ABSTRACT

A simple three-dimensional volcanic plume model that incorporates the effects of moisture and ambient wind is presented and used iteratively to refine the source mass flux, obtained from an empirical relationship between the plume rise height and source mass flux, according to the prevailing atmospheric conditions. It is applied to the eruption of Eyjafjallajökull in May 2010 using realistic atmospheric profiles appropriate to the time of the eruption. It is shown that the ambient wind has the largest effect on the refined source mass flux with moisture playing a secondary though still important role. It is also shown that significant differences in the values of the revised source mass fluxes can arise when the realistic atmospheric profiles are approximated by idealised profiles. The revised source mass flux is used to initialise a long-range dispersion model of ash in the atmosphere.

1. Introduction

It is well known (e.g. Woods, 1993; Glaze et al., 1997; Bursik, 2001; Mastin, 2007; Tupper et al., 2009) that the state of the atmosphere can have a significant impact on the rise height attained by volcanic plumes. In particular, weak eruptions can be strongly affected by the ambient wind and moisture can play a significant role in enhancing the growth of eruption columns especially in the tropics. Estimates of the source mass flux from empirical relationships with the observed rise height such as those proposed by Sparks et al. (1997, §5.2) and Mastin et al. (2009) do not take explicit account of the state of the atmosphere at the time of the eruption. This accounts for some of the considerable uncertainty in these empirical estimates. For example, if the volcanic plume is strongly bent over by the ambient wind then using one of the empirical relationships quoted above is likely to lead to an underestimate of the source mass flux. Conversely, moisture can add significantly to the energy of a volcanic plume via latent heating and so can potentially lead to an overestimate of the source mass flux.

Here, a simple three-dimensional plume model, that includes both moisture and ambient wind, is applied iteratively to an initial estimate of the source mass flux to produce a revised value using realistic atmospheric profiles. The initial estimate is obtained from the empirical relationships between the observed rise height and source mass flux described above. Results are presented for a short period of the Eyjafjallajökull eruption of 2010 in mid-May of that year as considered by Devenish et al. (2012). It will be shown that in this case the ambient wind plays an important role in the revised values of the source mass flux with moisture largely playing a secondary but not insignificant role. This is to be expected for a relatively weak extra-tropical eruption. Of course, using a simple plume model to revise the source mass flux according to atmospheric conditions at the time of the eruption assumes that the emitted ash rises to the same observed height as any gaseous material including water vapour.

The paper is organised as follows. The plume model is presented in the next section and is used in Section 3 to refine the initial estimated mass flux at the source according to the prevailing atmospheric conditions. The sensitivity of the revised values to changes in some of the parameters is also considered in this section. In Section 4 the revised values of the source mass flux are used to initialise the Met Office's operational dispersion model.

2. The moist plume equations in a crosswind

The model that will be used in Section 3 to revise the source mass flux according to ambient conditions is based on models by Woods (1988, 1993), Webster and Thomson (2002) and Devenish et al., (2010b) and combines the effects of moisture and the ambient wind together in a three-dimensional model. The governing equations are given by

$$
\begin{aligned}
\frac{dM_z}{ds} &= \left(\rho_a - \rho_p\right) g\pi b^2 \\
\frac{dM_i}{ds} &= -Q_m \frac{dU_i}{ds} \quad i = x, y \\
\frac{dH}{ds} &= \left(\left(1-q_v^a\right)c_{pd} + q_v^a c_{pv}\right) T_a \frac{dQ_m}{ds} - gQ_m \frac{\rho_a}{\rho_p}\frac{w_p}{v_p} \\
&\quad + \left[L_{v0} - 273\left(c_{pv} - c_{pl}\right)\right] \frac{dQ_l}{ds} \\
\frac{dQ_m}{ds} &= E \\
\frac{dQ_t}{ds} &= Eq_v^a
\end{aligned}
\qquad (1)
$$

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where $Q_m = \rho_p \pi b^2 v_p$ is the mass flux; $M_z = Q_m w_p$ is the vertical momentum flux; $M_i = (u_{pi} - U_i)Q_m$ are the horizontal momentum fluxes relative to the environment for $i = x, y$; $H = c_{pp} T_p Q_m$ is the enthalpy flux; $Q_t = Q_m n_t$ is the total moisture flux; and s is the distance along the plume axis. The meaning of the other symbols is given in Appendix A. The momentum-flux equations, (1a) and (1b), can be derived from the steady-state Navier–Stokes equations. In the derivation of these equations it is assumed that viscous forces can be neglected and that the pressure within the plume is equal to the ambient hydrostatic value. The mass-flux equation, (1d), can be derived from the continuity equation; (1e) can be derived from the equation for the conservation of water vapour and liquid water. For further details see e.g. Weil (1974), Weil (1988, p. 159) and Linden (2000). It is also assumed that there is no ambient liquid water and that there is no source liquid water flux. The model is restricted to phase changes between water vapour and liquid water though conceptually the extension to include ice is not precluded. An outline of the derivation of (1c) is given in Appendix B along with a discussion of the simplifying assumptions that underlie the derivation.

The lowest part of the eruption column is commonly referred to as the gas-thrust region (e.g. Woods, 1988; Sparks et al., 1997) in which the plume density exceeds the ambient density and the plume is driven by the momentum flux at the source rather than the buoyancy flux. In this region the entrainment rate, E, depends on both the ambient and plume densities (e.g. Morton, 1965):

$$E = 2\pi b \sqrt{\rho_a \rho_p}\, u_e \tag{2}$$

where u_e is the entrainment velocity. As the plume rises, sufficient heat may be transferred from the particulate material to the plume gas (assuming that the gas–solid mixture is approximately in thermal equilibrium) to allow the plume to rise due to buoyancy. Once $\rho_p \lesssim \rho_a$, Eq. (2) reduces to the familiar form $E = 2\pi b \rho_a u_e$ (e.g. Woods, 1988; Mastin, 2007).

It is commonplace to assume that there are two entrainment mechanisms in a crosswind (see e.g. Hoult et al., 1969; Hoult and Weil, 1972; Webster and Thomson, 2002; Devenish et al., 2010b), one due to velocity differences normal to the plume axis and the other due to velocity differences parallel to the plume axis and that the two mechanisms are additive. Devenish et al. (2010b) suggested that this additive entrainment assumption be an l^m-norm:

$$u_e = \left((\alpha|\Delta \mathbf{u_s}|)^{\mathbf{m}} + (\beta|\Delta \mathbf{u_n}|)^{\mathbf{m}} \right)^{1/m} \tag{3}$$

where $\Delta\mathbf{u}_s$ and $\Delta\mathbf{u}_n$ are the components of the relative velocity parallel to and perpendicular to the plume axis respectively, α and β are the entrainment coefficients associated with each entrainment mechanism and $m \geq 1$ is a tunable parameter. Throughout this study we take $\alpha = 0.1$ and $\beta = 0.5$ which are consistent with previous studies (see e.g. Hoult and Weil, 1972; Briggs, 1984; Devenish et al., 2010a, 2010b). For a source buoyancy flux F_0 and an atmosphere with (constant) buoyancy frequency, N, and (constant) wind speed, $\overline{U}$, the dimensionless wind speed $\tilde{U} = \overline{U}/(F_0 N)^{1/4}$ characterises the relative importance of the ambient wind speed compared with the vertical velocity of the plume. Here, N is obtained from a least-squares fit to the potential temperature profile over the depth of the plume (above the volcano summit) and $\overline{U}$ is the average wind speed over the same depth. In reality the source buoyancy flux is negative; here F_0 is taken to be an effective buoyancy flux once sufficient heat has been transferred to the gas phase to ensure a positive buoyancy flux. In the weak-wind limit, $\tilde{U} \ll 1$, the first term on the right-hand side of Eq. (3) dominates. When $\tilde{U} \gg 1$ the plume becomes bent-over and the second term on the right-hand side of Eq. (3) dominates. In both asymptotic limits u_e is independent of m; the dependence on m is at its most sensitive for $\tilde{U} = O(1)$. Devenish et al. (2010b) found that $m = 3/2$ gave the best agreement with large-eddy simulations of buoyant plumes in a crosswind and field observations.

The plume density is given by

$$\frac{1}{\rho_p} = \frac{n_g}{\rho_g} + \frac{1 - n_g - n_l}{\rho_s} + \frac{n_l}{\rho_l} \tag{4}$$

where the symbols are defined in Appendix A. Above the lower part of the plume, the volume fraction of ash (and any liquid water) is sufficiently small that $\rho_p \approx \rho_g / n_g$. The mass fraction of gas, n_g, can be derived from

$$\left(1 - n_g - n_l\right) Q_m = \left(1 - n_g^0\right) Q_m^0$$

where a superscript '0' indicates the value at the source and no fallout of either solid material or liquid water is assumed. The gas density (which includes both dry air and water vapour) is given by

$$\rho_g = \frac{p_a}{R_p T_p}$$

where p_a is the ambient pressure and $R_p = q_v R_v + (1 - q_v) R_d$ is the bulk gas constant (Woods, 1993) for the plume in which R_v is the gas constant of water vapour and R_d is the gas constant of dry air. The bulk specific heat capacity is given by

$$c_{pp} = n_d c_{pd} + n_v c_{pv} + n_l c_{pl} + \left(1 - n_g - n_l\right) c_{ps}$$

where the symbols are defined in Appendix A along with the values of the specific heat capacities (which are assumed to be independent of temperature).

Liquid water condensate is produced whenever the water vapour mixing ratio, r_v, is larger than the saturation mixing ratio, r_s, that is, $r_l = \max(r_t - r_s, 0)$ where r_l is the liquid water mixing ratio and r_t is the mixing ratio of the total water content. This can be expressed in terms of the mass fractions of water as

$$n_l = max(n_t - n_d r_s, 0) \tag{5}$$

which clearly allows for the possibility that liquid water can evaporate due to entrainment of dry air. Analytical expressions for r_s, which is a function of the dry pressure, p_d, and the local temperature, T, can be derived from the Clausius–Clapeyron equation on making use of $r_s = \varepsilon e_s / p_d$ where e_s is the saturation vapour pressure and $\varepsilon = 0.62$ is the ratio of the molecular mass of water vapour to dry air. For $-35\ °C \leq T \leq 35\ °C$, a simpler expression is given by a modification of Tetens' empirical formula,

$$e_s = 6.112\, exp\left(\frac{17.65T}{T + 243.5}\right), \tag{6}$$

which is accurate to within 0.3% (Emanuel, 1994, p. 117). (Note that Eq. (6) requires that pressure be measured in hPa and T in degrees Celsius; to a good approximation $p_d \approx p_a$.) Of course, one would expect much higher temperatures in a volcanic plume but condensation is not expected to occur until temperatures within the range $-35\ °C \leq T \leq 35\ °C$ are encountered well above the plume source. Thus, for our purposes Eq. (6) remains appropriate. Throughout this study it is assumed that any liquid condensate that forms remains in the plume i.e. the total water content is conserved. In practice, n_l is calculated using a Taylor expansion of r_s about some reference temperature in order to avoid spurious oscillations in the moisture phases between successive integration steps.

Equations (1a–e) are solved numerically using a fourth-order Runge–Kutta method. A fixed integration step proportional to $F_0^{1/4} N^{-3/4} / w_0$ is used where w_0 is the exit velocity and $F_0^{1/4} N^{-3/4}$ is the typical rise height of a buoyant plume in the absence of a crosswind. The integration is