Available online at www.sciencedirect.com
ScienceDirect
International Journal of Sediment Research 29 (2014) 195-207
INTERNATIONAL JOURNAL OF SEDIMENT RESEARCH
www.elsevier.com/locate/ijsrc

Stochastic particle based models for suspended particle movement in surface flows

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Abstract

Modeling of suspended sediment particle movement in surface water can be achieved by stochastic particle tracking model approaches. In this paper, different mathematical forms of particle tracking models are introduced to describe particle movement under various flow conditions, i.e., the stochastic diffusion process, stochastic jump process, and stochastic jump diffusion process. While the stochastic diffusion process can be used to represent the stochastic movement of suspended particles in turbulent flows, the stochastic jump and the stochastic jump diffusion processes can be used to describe suspended particle movement in the occurrences of a sequence of extreme flows. An extreme flow herein is defined as a hydrologic flow event or a hydrodynamic flow phenomenon with a low probability of occurrence and a high impact on its ambient flow environment. In this paper, the suspended sediment particle is assumed to immediately follow the extreme flows in the jump process (i.e. the time lag between the flow particle and the sediment particle in extreme flows is considered negligible). In the proposed particle tracking models, a random term mainly caused by fluid eddy motions is modeled as a Wiener process, while the random occurrences of a sequence of extreme flows can be modeled as a Poisson process. The frequency of occurrence of the extreme flows in the proposed particle tracking model can be explicitly accounted for by the Poisson process when evaluating particle movement. The ensemble mean and variance of particle trajectory can be obtained from the proposed stochastic models via simulations. The ensemble mean and variance of particle velocity are verified with available data. Applicability of the proposed stochastic particle tracking models for sediment transport modeling is also discussed.

Key Words: Suspended sediment, Particle tracking model, Stochastic modeling, Diffusion process, Poisson jump process, Jump diffusion process, Extreme flows

1 Introduction

Extreme flows such as flash floods or tsunamis are defined as low probability and high consequence hydrologic events. Such flows normally occur during a short period of time, most likely would cause mass casualties and destruction and have a major impact on facilities and lifeline. Other extreme flows, such as large turbulent bursts or large flow perturbations, typically observed in natural or controlled flow environments, might pose significant impacts on particle movement in ambient flow. Abrupt changes brought by large turbulent bursts or large flow perturbations could induce bridge scour and significantly change the mechanism of particle movement in surface waters and thus the water quality such as sediment concentration and turbidity etc. Either type of extreme flows unavoidably brings in an abrupt change in the movement of sediment particles in ambient surface waters and should be carefully studied.

In most of the flow and sediment models, flow equations are solved first in a given time step assuming a negligible bed elevation change, then subsequently the sediment governing equations are solved. The process is repeated until all variables converge. In this type of models, the extreme flow is treated and modeled as a succession of step-wise steady flows or as continuous time flows as the model input or the boundary conditions to both the flow and sediment equations. The aforementioned approaches have been widely used in modeling transport of sediment particles and have provided a good foundation for particle movement in response to extreme flows (Man, 2007). On the other hand, other researchers (Demissie, 1996; Toda et al., 2005; Markus and Demissie, 2006) developed statistical models to illustrate the relationship between the annual sediment load and the flood induced sediment load based on available data. The statistical method, however, does not consider the transport mechanism of the particles in response to the extreme flows.

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Note: The original manuscript of this paper was received in Sept. 2012. The revised version was received in Mar. 2013. Discussion open until June 2015.

Recently, researchers started to incorporate the stochastic feature of hydrologic events into sediment transport modeling. Among others, Singer and Dunne (2004) coupled a stochastic hydrologic model with a sediment transport model to predict daily bed sediment transport rates. In spite of the aforementioned efforts, the understanding of the sediment particle movement in rivers in response to such events remains incomplete, particularly in the aspect of the stochastic behavior of particle movement.

Moreover, most of the existing modeling approaches for transport of sediment particles treat the given extreme flow event with a certain return period as a boundary condition to the deterministic flow and sediment governing equations when modeling transport of sediment particles under a given extreme flow condition. Most of deterministic modeling methods do not directly express random characteristics of sediment concentrations, loads and transport rates in the governing equations. Such information is critical to scientifically quantify effective risks of exceeding an established water quality standard due to extreme flow events. Furthermore, the frequency of occurrence of the extreme flow events cannot be explicitly accounted for in the evaluation of movement of sediment particles in existing modeling approaches. When an extreme flow event happens, the particle trajectory is changed as the surrounding flow field is subject to an abrupt disturbance as a result of the extreme flow occurrence. For example, in a flood prone water body, particles in suspension or on the channel bed and bank are subject to more frequent entrainment and deposition processes. The frequency of occurrence of the extreme flows thus plays an important role in developing a forecast model to analyze movement of particles in flows.

The objective of this paper is to propose a Lagrangian stochastic model of particle movement with an attempt to account for the uncertainty of abrupt particle movement associated with the occurrence of extreme flows. The practical outcome of this work is twofold: (i) to explicitly account for the probability of occurrence of the extreme events in the modeling of sediment transport, and (ii) to further study the impact of extreme flows on the particle movement as opposed to that of the base flow conditions. In the following section, a stochastic diffusion model, a Poisson jump model and a stochastic jump diffusion model are introduced. Subsequently, particle movement under different scenarios is presented to demonstrate the impact of the Poisson jumps of the particle movement in the presence of the extreme flows.

2 Stochastic model for particle transport

A stochastic process is defined as a spatial or temporal process involving probability (Yen, 2002). To describe the stochastic properties of entities, researchers in various fields have employed stochastic differential equation (SDEs) such as Ito processes or Langevin equations (Verwoerd and Kulashiri, 2003; Dean and Russel, 2004). Since the movement of sediment particle is intrinsically stochastic, an SDE is appropriate to depict uncertain movement of particles. There are several studies on sediment transport from the stochastic viewpoint. Most of the studies apply probabilistic inputs or parameters to deterministic governing equations (Kleinhans and van Rijn, 2002; Sharma and Kavvas, 2005). In this respect, the stochastic governing equations containing random processes are expected to be widely used without many restrictions on inputs or parameter distribution.

2.1 Modeling turbulence as a Wiener process

A Wiener process or Brownian motion process can be extended from the perspective of limiting process of a random walk. The random walk can be expressed as follows. Let X_i be independently and identically distributed, the size of the ith step with $\Pr(X_i=k) = p$ and $\Pr(X_i=-k) = 1-p = q$, where k is the size of a step. With probability p the walk is toward the positive direction. On the other hand, with probability q the walk is toward the negative direction. If Z_n denotes the position of the random walk after n steps on a Z-n plane, the stochastic process $\{Z_n, n\geq 0\}$ is called a random walk process. $Z(t)=X_1+\ldots+X_{[t/\Delta t]}$ for continuous time where an interval of length t is divided into subintervals of length Δt. Wiener processes inherit the properties of being the limiting process of a random walk, namely (i) $Z(0)=0$, (ii) Z has stationary and independent increments, and (iii) for every $t \geq 0$, $Z(t) \sim N(\bar{u}t, \sigma^2 t)$ where $\bar{u}$ is the mean and σ^2 is the variance of the normal distribution. The sample path of Wiener process Z(t) is a continuous function of t.

The random variable $W(t)$ is a standard Brownian motion or a standard Wiener process in continuous time $0 \leq t \leq T$, which satisfies the following conditions (Higham, 2001; Oksendal, 1998)

(1) $W(0) = 0$ with probability 1.

(2) If $0 < s < t < T$, then the random variable $\Delta W = W(t) - W(s)$ is normally distributed with a zero mean and a variance (t – s), and satisfies

$$\frac{\Delta W}{\sqrt{t-s}} \sim N(0,1)$$

where $N(0,\ 1)$ denotes a normally distributed random variable with a zero mean and a unit variance.

(3) If $0 < s < t < u < v < T$, $\Delta W_1 = W(t) - W(s)$ and $\Delta W_2 = W(v) - W(u)$, then ΔW_1 and ΔW_2 are independent.

The Wiener process for particle movement in a turbulent flow is expected to be suitable for various situations