Journal of Volcanology and Geothermal Research 253 (2013) 114–130

Contents lists available at SciVerse ScienceDirect

Journal of Volcanology and Geothermal Research

journal homepage: www.elsevier.com/locate/jvolgeores

Analytical model of surface uplift above axisymmetric flat-lying magma intrusions: Implications for sill emplacement and geodesy

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ARTICLE INFO

Article history:
Received 21 July 2012
Accepted 3 December 2012
Available online 20 December 2012

Keywords:
Sills
Laccoliths
Analytical model
Axi-symmetric
Ground deformation
Mogi

ABSTRACT

In this paper, we develop a new axisymmetric analytic model of surface uplift upon sills and laccoliths, based on the formulation of a thin bending plate lying on an elastic foundation. In contrast to most former models also based on thin bending plate formulation, our model accounts for (i) axi-symmetrical uplift, (ii) both upon and outside the intrusion. The model accounts for shallow intrusions, i.e. the ratio $a/h > 5$ where a and h are the radius and depth of the intrusion, respectively. The main parameter of the model is the elastic length l, which is a function of the elastic properties of the bending plate and of the elastic foundation. The model exhibits two regimes depending on the ratio a/l. When $a/l < 5$, the uplift spreads over a substantial domain compared to that of the intrusion. In contrast, when $a/l > 5$, the uplift is mostly restricted upon the intrusion. When the elastic foundation is very stiff, our model converges towards that of a clamped plate. We provide, as supplementary material, a Matlab function that calculates the surface uplift from the set of system and control parameters. We discuss three possible applications of our model: (i) The model can be used to describe sill propagation by introducing a propagation criterion. For realistic values, our model reproduces well the behavior of horizontal intrusions simulated in experiments; (ii) The model can also be used to compute the critical size of saucer-shaped sills. It shows, for instance, that a soft elastic foundation favors the horizontal spreading of sills before they form inclined sheets; (iii) We show that the classical Mogi point source model cannot be used to constrain sill properties from the surface uplift. We thus propose that our model can be used as a valuable alternative to both simple analytical models like Mogi's and more complex numerical models used to analyze ground deformation resulting from sill intrusions in active volcanoes.

1. Introduction

Surface deformation in active volcanic systems is generally assumed to reflect the dynamics of magma intrusion and transport at depth. Modern monitoring techniques allow good measurements of the deformation of volcanic edifices before, during and after an eruption (Cayol and Cornet, 1998; Froger et al., 2001, 2007; Fukushima et al., 2010; Galland, 2012). A good example is the Eyjafjallajökull Volcano, Iceland, which has been monitored for more than a decade using InSAR and GPS data (Fig. 1a). These data allowed to detect the onset of its unrest (Pedersen and Sigmundsson, 2006), and overall the premises of the 2010 eruption that caused massive disruptions in the air traffic in Europe (Sigmundsson et al., 2010).

The mechanical analysis of surface deformation patterns is commonly used to constrain the geometry and dynamics of the magma plumbing systems *a posteriori* (Cayol and Cornet, 1998; Fukushima et al., 2005; Masterlark, 2007). One of the first attempts in analyzing surface deformation on active volcanoes has been performed by Mogi (1958), who developed an analytical solution of surface deformation induced by a small spherical over-pressured magma reservoir. The solution of Mogi is valid when the size of the magma reservoir is very small compared to its depth, i.e. when $a_m/h_m << 1$, where a_m and h_m are the radius and the depth of the center of the reservoir, respectively.

Although the so-called Mogi point source solution provides good fits with data monitored on some active volcanoes, recent studies show that (1) many magma reservoirs do not consist of spherical chambers but exhibit a flat-lying shape (e.g. Amelung et al., 2000; Fialko et al., 2001a, 2001b; Pedersen and Sigmundsson, 2006; Chang et al., 2007; Sigmundsson et al., 2010; Woo and Kilburn, 2010), and (2) the roof of the reservoir can be very shallow (<3 km) (McTigue, 1987; Brandsdóttir and Menke, 1992; Gudmundsson et al., 1994; Brandsdóttir et al., 1997; Brandsdóttir and Menke, 2008; Elders et al., 2011), such that the assumption $a_m/h_m << 1$ is not satisfied. The Mogi point source solution is thus not applicable in these conditions, and a more relevant analytical solution is needed to interpret surface deformation data.

Surface deformation has not only been observed in active volcanoes as a passive consequence of shallow magma emplacement, but it can exert an active mechanical feedback on the emplacement of laccoliths and

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0377-0273/$ – see front matter
http://dx.doi.org/10.1016/j.jvolgeores.2012.12.006

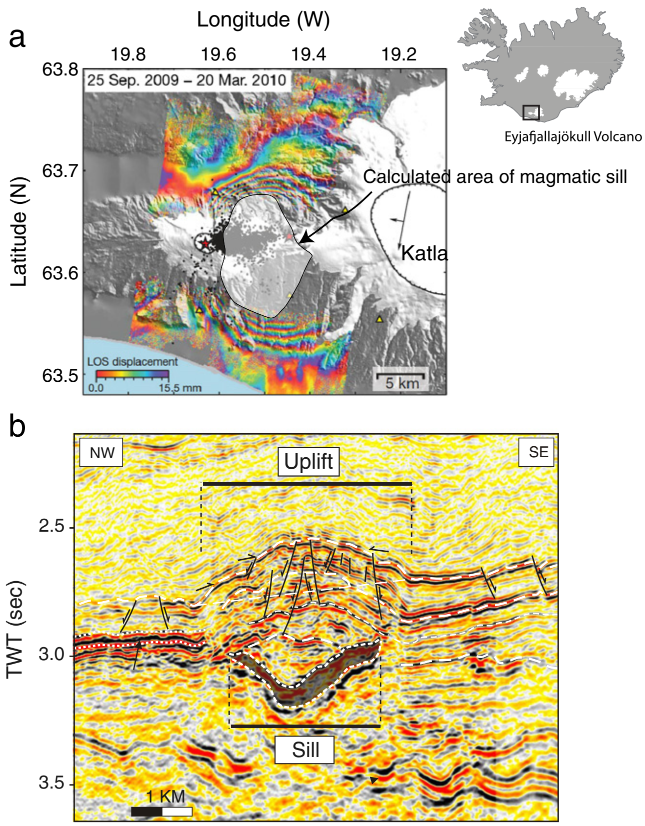

Fig. 1. Geological examples of uplift due to sill emplacement. a. Satellite image of Eyjafjallajökull Volcano, Iceland, showing InSAR and seismic data during the Spring 2010 eruption (after Sigmundsson et al., 2010). The data were monitored between 25 September 2009 and 20 March 2010. They were related to pre-eruptive period, which corresponded to the emplacement of a ~10 km large sill at a few kilometers depth (transparent white surface; Fig. 3e of Sigmundsson et al., 2010). Black arrows show the satellite flight path (downward arrow) and look directions (leftward arrow). Black dots show earthquake epicenters during this period. The red stars locate the eruption localities. The yellow triangles locate GPS stations that monitored continuously flank deformation. The colored fringes represent ground displacement calculated from TerraSAR-X interferograms from descending satellite orbits. Each fringe corresponds to line-of-sight, i.e. distance from the satellite, change of 15.5 mm. The total displacement can thus be several tens of centimeters. Background is shaded topography. The rounded patterned line on the right of the image locates the caldera of Katla Volcano. Note that the uplifted area is at least twice wider than the underlying sill. b. Seismic profile illustrating the relationships between a saucer-shaped sill and the structure in its overburden, Rockall Basin, offshore Scotland (modified after Hansen and Cartwright, 2006b). Vertical scale is the time for seismic wave travel (in seconds). The profile shows that sill overburden is bent, forming a dome structure (uplift). This profile shows that the dome is about 1.3 times wider than the sill.

saucer-shaped sills (e.g. (Gilbert, 1877; Jackson and Pollard, 1990; Jackson, 1997; Malthe-Sørenssen et al., 2004; Polteau et al., 2008; Galland et al., 2009; Galerne et al., 2011). On seismic images and in the field, it can be observed that saucer-shaped sills, for instance, are closely associated with uplift and bending of their overlying strata, the inclined sheets being located under the edges of the uplifted area (Fig. 1b; Hansen and Cartwright, 2006a; Muirhead et al., 2012). This relationship has been interpreted as a result of the mechanical interaction between the bending of the overlying strata and the spreading of the sills: the differential uplift at the edges of the domes generates stresses that interact with the leading edges of the sills, which in turn are deflected towards the surface (Malthe-Sørenssen et al., 2004; Goulty and Schofield, 2008; Galland et al., 2009). This mechanism producing inclined sheets substantially contributes to magma ascent through sedimentary basins (Cartwright and Hansen, 2006; Muirhead et al., 2012).

In order to (1) better predict surface deformation due to the emplacement of shallow flat-lying intrusions and (2) better quantify how the bending of strata affects the emplacement of laccoliths and saucer-shaped sills, one needs to better constrain the mechanics of surface deformation. In this paper, we develop a new analytical model of surface deformation above shallow axially symmetric flat-lying intrusions. Our model is based on the theory of a thin bending plate lying on a deformable elastic foundation. After the theoretical development, we discuss the limitations and the effects of the parameters on the model. Subsequently, we discuss some geological applications, notably for sill propagation and for saucer-shaped sills. We also briefly discuss the potential application of our model to the analysis of ground deformation due to sill intrusion in active volcanoes. We provide the code of our model as a Matlab function available as supplementary material.

2. Existing solutions

Several models have been developed to calculate the deformation field associated with flat-lying intrusions. Because sills are sheet intrusions, they can be considered as horizontal fluid-filled cracks within an elastic medium. The strains in the elastic medium are considered to be small everywhere except close to the tip of the sill, and the formulation of the problem can be achieved using the theory of linear elasticity.

A first approach attempted to develop models tending toward a complete description of cracks in an elastic half-space. Sun (1969) described the deformation of a free surface above a fluid-pressurized crack by developing a 3D approximate solution of the vertical and horizontal displacements above a circular crack. To do that, he superimposed (i) the solution for the displacements due to a crack in an infinite elastic medium and (ii) an auxiliary stress function that satisfies the zero traction boundary conditions at the free surface. However, such superposition generates significant errors when the crack's radius-to-depth ratio a/h becomes greater than 1 (Fialko et al., 2001b). Pollard and Holzhausen (1979) developed the 2D equations that account for the surface deformation and the stress intensity factor at the tip of an arbitrarily oriented crack contained in an elastic half-space. More recently, Fialko and Simons (2001) derived the axi-symmetrical solutions for the stress and displacements associated with a horizontal circular crack in an elastic half-space. Although these approaches are powerful tools, obtaining the solutions requires numerical integrations, which can be tricky to implement, like for most analyses of elastic layers under stress (see e.g. Scheibert et al., 2009).

A second, classical approach for describing surface uplift due to a sill or a laccolith is based on thin elastic plate theory (Timoshenko and Woinowsky-Krieger, 1959). This theory can easily be adapted to investigate the deflection of sedimentary strata above a magma-filled horizontal sill or laccolith (Pollard and Johnson, 1973; Scaillet et al., 1995; Goulty and Schofield, 2008; Michaut, 2011). This theory accounts only for shallow sills, i.e. the radius of the crack a is large with respect to its depth h (typically $a/h > 5$), which is the case for many sills. This approach, developed both in 2D and 3D, has been extensively used and generally accepted, such that it is presented as a classical model in textbooks (Turcotte and Schubert, 2002) and is used in the mechanics community (Murdoch, 1993a,b,c, 2002; Bunger, 2005; Bunger and Cruden, 2011).

The application of thin plate theory alone to sills and laccoliths takes into account the weight of the overburden (q_0), heterogeneous magma pressure distributions in the intrusion ($P(x)$), and the mechanical layering of the overburden (Pollard and Johnson, 1973). Very recently, the thin plate formulation has been coupled with the equations for viscous fluid flow to model the emplacement of viscous magma into sills and laccoliths (Bunger and Cruden, 2011; Michaut, 2011). In addition, it has been extended to derive a simple criterion for the upward propagation of saucer-shaped sills (Goulty and Schofield, 2008). Nevertheless, the formulations developed in these papers assume that the bending plate is clamped to a rigid foundation at the tips of the intrusions (Pollard and