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Flow resistance of gravel bed channels

Wen C. WANG[1] and David R. DAWDY[2]

Abstract
Existing resistance formulas produce a wide range of friction-factor estimates for gravel bed streams. The purpose of this paper is to develop a reliable resistance formula in terms of the Darcy-Weisbach friction factor *f*. Published data were screened and used to establish the formula. The existing formulas have considered that *f* is a function of relative roughness D_{84}/R only, where *R* is the hydraulic radius and D_{84} is the particle size referred to the intermediate diameter that equals or exceeds that of 84 percent of bed sediments. In this paper, *f* is considered as a function of Froude number in addition to the relative roughness. *f* for $D_{84}/R>1$ displays a different trend than that for $D_{84}/R<1$ perhaps due to the invalid assumption of a logarithmic velocity distribution for $D_{84}/R>1$. An *f* formula for $D_{84}/R<1$ has been established.

Key Words: Flow resistance, Darcy-Weisbach friction factor, Relative roughness, Relative submergence, Gravel bed channels, Open channels

1 Introduction

Accurate estimation of flow resistance is important for the hydraulic analysis of streams. Resistance coefficients can be better estimated for lowland alluvial channels than upland gravel bed streams using existing formulas. For gravel bed streams, existing formulas generally produce a wide range of estimates. It is the purpose of this paper to develop a reliable resistance formula in terms of the Darcy-Weisbach friction factor for gravel bed streams. In doing so, published data and formulas were first reviewed and screened. A new formula was then established based on the screened data and compared with existing formulas. At least six different equations are in the literature which are based on the Darcy-Weisbach formula. In order to set the stage for the new formula, the several present formulas are presented. Then the screening criterion is described and used to explain some of the problems with previous analyses.

2 Flow resistance coefficients

Flow resistance may be represented by the Darcy-Weisbach friction factor *f* defined below:

$$f = h_L \frac{d}{L}\frac{2g}{V^2} \tag{1}$$

where h_L= head loss, L = pipe length, d = pipe diameter, V = average velocity, and g = gravitational acceleration. The formula was originally developed for pipe flows but can be applied to open channel flows. Equation1 can be transformed to the following form:

$$\frac{1}{\sqrt{f}} = \frac{1}{\sqrt{8}}\frac{V}{u_*} \tag{2}$$

where u_* is the shear velocity defined as $u_* = \sqrt{\tau_o/\rho}$ with τ_o and ρ being boundary shear stress and the density of water, respectively. The Darcy-Weisbach friction factor *f* may be related to the commonly used Manning's roughness coefficient *n* as follows:

$$\frac{1}{\sqrt{f}} = \frac{1.486}{n}\left(\frac{R^{1/6}}{\sqrt{8g}}\right) \tag{3}$$

where *R* is the hydraulic radius in ft and g = 32.2 ft sec^{-2}.

The Darcy-Weisbach friction factor was experimentally investigated for pipe flows by Nikuradse (1933). The

[1] President, Multech Engineering Consultants, Inc. 1650 Zanker Road, Suite 210, San Jose, CA 95112,
E-mail: wang@multecheng.com
[2] Chief Hydrologist, Multech Engineering Consultants, Inc. 1650 Zanker Road, Suite 210, San Jose, CA 95112,
E-mail: dawdy@multecheng.com

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experiments were conducted on smooth pipes and on rough pipes internally coated with sand grains of uniform size. The results were presented in graphical form in the well-known Moody diagram. Prandtl (1926) and von Karman (1930) established the following resistance formula for turbulent flows in fully rough pipes:

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{k_s}{3.7d}\right) \tag{4}$$

The friction factor determined from the Moody diagram can be applied to turbulent open channel flows with uniform bed sediments, provided that the channels are without bed forms and bank vegetation and the effect of the free surface is negligible. The application can be made by substituting the pipe diameter d with the hydraulic radius R (namely, $d = 4R$) and taking a characteristic grain diameter as k_s in Eq. 4 as follows:

$$\frac{1}{\sqrt{f}} = 2.34 - 2\log\left(\frac{k_s}{R}\right) \tag{5}$$

3 Darcy-Weisbach friction factor for natural open channel flows

Bed sediments of natural channels are generally not uniform in size. This type of boundary results in higher flow resistance than that predicted by the Moody diagram because of formation of eddies behind large bed particles. To account for this effect, particle diameters greater than the mean diameter are often adopted as k_s in resistance formulas as proposed by some investigators. In order to understand the problem, frequently used resistance formulas for natural open channels are described below:

A. Leopold and Wolman (1957) developed an empirical formula for the friction factor as follows:

$$\frac{1}{\sqrt{f}} = 1.0 - 2\log\left(\frac{D_{84}}{R}\right) \tag{6}$$

where D_{84} is the particle size, referred to the intermediate diameter that equals or exceeds that of 84 percent of bed sediments.

B. Limerinos (1970) used field data from 11 sites in California streams to establish the following formula for estimating Manning's n:

$$n = \frac{0.0926R^{1/6}}{1.16 + 2\log(R/D_{84})} \tag{7}$$

where $R$and D_{84} are in ft. Bed materials at the gauging sites ranged from gravel to boulder. The sites were relatively free of the flow-retarding effects associated with irregular channel plan-form and bank vegetation. Equation 7 can be combined with Eq. 3 to become:

$$\frac{1}{\sqrt{f}} = 1.16 - 2\log\left(\frac{D_{84}}{R}\right) \tag{8}$$

C. Hey (1979) used data collected from 21 sites of gravel bed channels in the United Kingdom to establish the following formula:

$$\frac{1}{\sqrt{f}} = 2.03\log\left(\frac{aR}{3.5D_{84}}\right) \tag{9}$$

where a is a coefficient. The form of the above formula reflects Hey's argument that the height of representative roughness elements is 3.5 times D_{84}. The formula can be rearranged to the following form:

$$\frac{1}{\sqrt{f}} = -1.10 + 2.03\log(a) - 2.03\log\left(\frac{D_{84}}{R}\right) \tag{10}$$

Hey suggested that coefficient "a" varies with channel cross-sectional geometry and ranges from 11.1 for very wide open channels to 13.4 for circular pipes. The above formula can be rearranged for wide open channels with a =11.1 as follows:

$$\frac{1}{\sqrt{f}} = 1.02 - 2.03\log\left(\frac{D_{84}}{R}\right) \tag{11}$$

D. Bray (1979) used data collected from 67 sites in natural gravel be drivers in Alberta, Canada to establish the following formula:

$$\frac{1}{\sqrt{f}} = 1.26 - 2.16\log\left(\frac{D_{90}}{d}\right) \tag{12}$$

where d is the average flow depth and D_{90} is the particle size that equals or exceeds that of 90 percent of bed sediments.

E. Griffiths (1981) used data collected from gravel bed rivers in New Zealand to establish the following relationship:

$$\frac{1}{\sqrt{f}} = 0.76 - 1.98\log\left(\frac{D_{50}}{R}\right) \tag{13}$$