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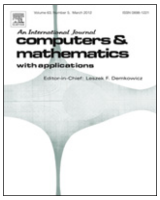

Stress field around a strike-slip fault in orthotropic elastic layers via a hypersingular integral equation

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ABSTRACT

This paper studies a vertical strike-slip fault in a layered orthotropic elastic medium perfectly bonded to a rigid foundation. Using elastic analysis of antiplane shear deformation, the strike-slip fault is converted to a mode III crack problem and the associated boundary value problem is reduced to triple series equations by expanding elastic displacement as Fourier series. A hypersingular integral equation for elastic displacement jump across the fault is then derived. An approximate solution of the displacement jump is constructed in terms of Chebyshev polynomials. Full elastic displacement and stress distribution in a layered medium are obtained. The formulae for calculating stress intensity factors near the crack tips are presented for any loading. The effects of stress drop and coefficient of friction on the stress field around a strike-slip fault are discussed.

1. Introduction

An earthquake is often understood as a sudden dynamic propagation of a fault when stored energy near the fault accumulates to its critical value. Cracks and dislocations are two typical defects, which are originally used in solids to account for the mechanism of brittle failure and plastic deformation of structures. The crack and dislocation method is also a powerful analytical tool for better understanding the physical nature of an earthquake, and has been widely used in this field [1,2]. Great progress has been made in theoretical research [3–5]. Strike-slip faults are a class of typical faults, and the corresponding strike-slip deformation is commonly simulated as antiplane shear deformation.

Within the framework of fracture mechanics, antiplane shear cracks embedded in a homogeneous isotropic elastic half-space have been extensively studied. However, strike-slip faults occur in a tectonic structure, which has apparent anisotropy of the geological constituents. In particular, such heterogeneous structural characteristics have pronounced difference relying on the depth of position. As a result, a layered medium is more effective than an elastic half-space. Turcotte and Spence [6] treated a strike-slip fault between two lithospheric plates and calculated the energy stored in the layered medium. Furthermore, Rowshandel and Nemat-Nasser [7] used this model to illuminate the effects of geometry and material properties on fault instability and rupture initiation. By using a conformal mapping, a closed-form solution has been derived in [8] for an edge crack subjected to appropriate boundary conditions. In the above-mentioned researches, for the simplicity of mathematical analysis, the strike-slip fault is assumed to be terminated at the bottom surface. In particular, the top surface is traction-free and the bottom is either traction-free or uniform displacement or strain. A more realistic case is a layered medium bonded to an elastic half-space. Along this line, the effects of the upper elastic layer on the quasi-static growth of strike-slip faults have been analyzed [9–15]. For a viscoelastic medium lying in the bottom, a vertical strike-slip fault

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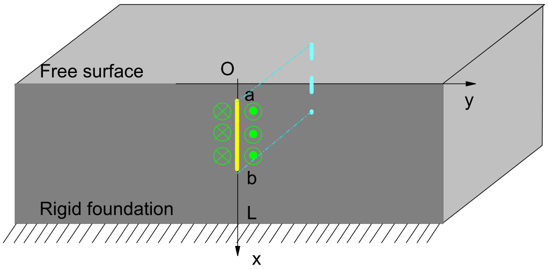

Fig. 1. Schematic of a strike-slip fault in an orthotropic elastic layer welded to a rigid foundation.

embedded in an elastic layer was investigated in [11]. For a vertical finite screw dislocation in a layered medium bonded to an elastic half-plane, Rybicki [12] used an image method to derive the expressions for the surface displacement in the form of an infinite series. For the same problem, Du et al. [13] applied the perturbation method to obtain several lower-order perturbation solutions with satisfactory accuracy. Using the elementary solutions due to a screw dislocation, Bonafede et al. [14] further analyzed an antiplane crack in a layered half-plane and derived numerical results of stress distribution by using the singular integral equation method as well as the dislocation pileup model. Based on the dislocation pileup model, Savage [16] developed a model of strain accumulation from an isolated screw dislocation in an elastic half-space with the Burgers vector increasing at the rate of relative plate motion. Singh and Rani [17,18] analyzed 2-dimensional lithospheric deformation induced by a strike-slip fault. Maccaferri et al. [19] gave a numerical analysis of the propagation of a fluid-filled crack in layered elastic media. Hirano and Yamashita [20] studied a fault lying along and intersecting a bimaterial interface and obtained static stress field around faults.

In this paper, we consider an orthotropic elastic layer with a vertical strike-slip fault. The top surface of the layer is free and the bottom is perfectly bonded to a rigid foundation. Under the action of arbitrary antiplane shear loading, the whole elastic field of stresses in the layer is obtained. Moreover, the formula for calculating stress intensity factors near the fault tip are analytically derived.

2. Statement of the problem

Consider a layered orthotropic elastic medium perfectly bonded to a rigid foundation. The top surface of the layer is traction-free. A vertical strike-slip fault is embedded in this layer and located at $a \le x \le b, y = 0$ and the thickness of the layer is denoted as H, then $0 \le a < b \le H$ (see Fig. 1). When $a = 0$ or $b = H$, the strike-slip fault reduces to an edge fault or a crack. Conversely, it is called an internal crack. Since the problem is related to antiplane shear deformation, there is only out-of-plane displacement $w\,(x, y)$, which satisfies the following governing differential equation

$$C_{55}\frac{\partial^2 w}{\partial x^2} + C_{44}\frac{\partial^2 w}{\partial y^2} = 0, \tag{1}$$

where C_{44}, C_{55} are two elastic constants. When $C_{44} = C_{55}$, the orthotropic layer reduces to isotropic elastic layer, and in this case $C_{44} = C_{55}$ is often denoted as shear modulus μ. Once w is determined, the stresses can be evaluated by the following relations:

$$\tau_{xz} = C_{55}\frac{\partial w}{\partial x}, \qquad \tau_{yz} = C_{44}\frac{\partial w}{\partial y}. \tag{2}$$

At the top and bottom surfaces, we have the following boundary conditions

$$\tau_{xz}(0, y) = 0, \qquad w(H, y) = 0. \tag{3}$$

For crack problems, of much interest is the singular elastic field disturbed by a crack which is directly related to the stored energy. Here let us consider the case of arbitrary antiplane shear loading, namely

$$\tau_{yz}(x, 0) = -\tau_0(x), \quad a < x < b, \tag{4}$$

where $\tau_0(x)$ is the prescribed stress depending on the stress drop and tectonic stress.

3. Derivation of equation

First, the symmetry of the problem allows us to write $w\,(x, y) = -w\,(x, -y)$ and $w\,(x, 0) = 0$ for $0 < x < a$ and $b < x < H$. Due to the cause of symmetry, only the elastic field in the $y \ge 0$ is solved here. Similar to the treatment in [21], solving Eq. (1) subject to the conditions in (3) one may take the displacement as the following cosine series

$$w(x, y) = \sum_{n=0}^{\infty} A_n \exp\left[-(2n+1)\,\alpha\beta y\right]\cos\left[(2n+1)\,\beta x\right], \quad 0 \le x \le H, \tag{5}$$