Computers and Mathematics with Applications 66 (2013) 2400–2424

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

The MFS for the solution of harmonic boundary value problems with non-harmonic boundary conditions

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ARTICLE INFO

Article history:
Received 21 May 2013
Received in revised form 26 August 2013
Accepted 1 September 2013

Keywords:
Method of fundamental solutions
Laplace equation
Non-harmonic boundary conditions

ABSTRACT

We investigate applications of the method of fundamental solutions (MFS) for the numerical solution of two-dimensional boundary value problems in complex geometries, governed by the Laplace equation and subject to Dirichlet boundary conditions which are not harmonic. Such problems can be very challenging because of the appearance of boundary singularities. We consider several ways of choosing the boundary collocation points as well as the source points in the MFS. We show that with an appropriate such choice the MFS yields highly accurate results.

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1. Introduction

Over the past two decades, the method of fundamental solutions (MFS) has been continuously gaining attention in the science and engineering community due to a series of the developments in this area [1–3]. One of the main attractions of the MFS is its simplicity. However, despite its effectiveness and simplicity, there are still some theoretical and practical issues to be satisfactorily addressed. One of the main issues yet to be resolved is the choice of the source location. In the early stages of the development of the MFS, there were two major approaches for choosing the source points: the fixed (or static) approach and the adaptive (or dynamic) approach. Readers are referred to [1,2,4,5] and references cited therein for more details. In this paper we will focus on the fixed approach which, in general, has been more popular in real-life applications due to its practicality and simplicity. The adaptive approach [6] involves the solution of a system of nonlinear equations, even in the case of linear problems, a fact which makes it highly impractical, especially when dealing with large scale problems.

It is known that when the boundary shape and the boundary data are sufficiently smooth, the MFS converges rapidly. As such, the location of the source points has little effect on the accuracy of the MFS. In such cases, for simplicity, the fictitious boundary is usually taken to be a circle of large radius [7]. However, in real-life applications, where the boundary shape and data might not be as smooth, practitioners often encounter difficulties in selecting the source location. In recent years, various techniques have been proposed to deal with this issue. Some of these techniques are indeed very elegant and sophisticated, but lose the major attractive feature of the MFS which is its simplicity and hence ease of implementation. This might be the reason for which the early adaptive approach and its variants have not been very widely used. It is for these reasons that we will concentrate on proposing simple and yet effective algorithms to keep the implementation of the MFS as simple as possible.

The aim of this paper is to investigate how to efficiently select, not only the source location, but also the boundary collocation point distribution, and the effect of this selection on the accuracy of the MFS, for the solution of elliptic boundary

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http://dx.doi.org/10.1016/j.camwa.2013.09.004

value problems in complex geometries. In particular, we consider problems governed by the Laplace equation and which are subject to Dirichlet boundary conditions which are not harmonic. Usually, the problems which are considered in order to test the accuracy of a method have boundary conditions constructed from a globally harmonic function and hence are piecewise harmonic on the boundary. In fact, if the boundary data come from a harmonic function without a singularity inside the domain, it is sufficient to interpolate by harmonic functions on the boundary, and there is no need to consider the partial differential equation. The application of the MFS to problems with non-harmonic boundary conditions has been the subject of a recent study by Schaback [8]. In particular, Schaback claims that when a boundary value problem governed by the Laplace equation is subject to piecewise harmonic boundary conditions, then it is preferable to use approximations with harmonic polynomials instead of the MFS. In contrast, when the boundary condition is not piecewise harmonic and the boundary of the domain of the problem under consideration contains re-entrant corners, it is preferable to use the MFS. In general, the approximation of the solution of a harmonic problem subject to a (piecewise) harmonic boundary condition in any geometry (with or without re-entrant corners) by the MFS method will be excellent. However, the presence of re-entrant corners in problems with non-(piecewise) harmonic boundary conditions will, in general, give rise to boundary singularities [9]. In the case of boundary curves which are analytic but include sharp curvature changes, such as amoeba-like curves, then one might expect singularities in the complement of the closure of the domain, but very close to the boundary as it occurs in the case of conformal mapping [10]. In the case of boundary singularities, it is possible to predict their general nature [11–14] and enrich the approximation by the appropriate singular functions. This approach has been applied with the MFS in several studies [15–18].

Let us first consider the claim that when dealing with harmonic problems with harmonic boundary conditions which are thus singularity-free it is preferable to use harmonic polynomials. This approach is what many authors call the Trefftz collocation method [19] and it is in a way natural to apply it to such problems. This approach, however, has a serious disadvantage with respect to the MFS which is the fact that the resulting systems are very poorly conditioned. One way of overcoming this problem, in certain cases, is to normalize the coefficients in the harmonic polynomial expansion [20,21], or to even use regularization techniques [22].

Regarding the application of the MFS to harmonic problems with non-(piecewise) harmonic boundary conditions in problems in complex geometries, we show that the distribution and placing of the collocation points and the sources plays a crucial role in the accuracy of the method. It is certainly not advisable to place the source points on a circle surrounding the domain of the problem under consideration but preferable to place them on a curve congruent to the boundary of the domain, with each source point placed at a fixed distance from the corresponding boundary collocation point. In addition, this distance needs to be kept small. Also, improved results can be obtained with the equal spacing of the boundary collocation points as this improves considerably the conditioning of the MFS matrices. Similar observations were registered when the method was applied to a simple geometry (a disk) subject to boundary conditions yielding boundary singularities.

It should be mentioned that the accuracy of the MFS for harmonic problems subject to non-harmonic boundary conditions and its relation to the so-called effective-condition-number has been the subject of some recent studies [23,24], see also [25,26].

In Section 2 we present the general problem we consider and the MFS formulation for its solution. The four approaches which involve the selection of the collocation and source points are described in Section 3. Five numerical problems with different boundary shapes are defined in Section 4. The results obtained using the four approaches on these five test problems are given in Section 5. Finally, some concluding remarks are provided in Section 6.

2. The problem and method of solution

Since the main ideas developed in this paper can be extended to other types of boundary conditions, we keep our presentation simple by only considering the two-dimensional Dirichlet boundary value problem for the Laplace equation

$$\Delta u = 0 \quad \text{in } \Omega, \tag{2.1a}$$

subject to the Dirichlet boundary condition

$$u = f \quad \text{on } \partial\Omega, \tag{2.1b}$$

where the function f is a not necessarily harmonic given function.

2.1. The method of fundamental solutions (MFS)

We seek an approximation to the solution of Laplace's equation (2.1a) as a linear combination of fundamental solutions of the form [1,2]

$$u_N(\boldsymbol{c}, \boldsymbol{\xi}; \boldsymbol{x}) = \sum_{k=1}^{N} c_k\, G(\boldsymbol{\xi}_k, \boldsymbol{x}), \quad \boldsymbol{x} \in \overline{\Omega}, \tag{2.2}$$

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